The Capacity of Wireless Communication Systems Can Be Substantially Increased by the Use of Antenna Diversity

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Abstract

For a broad class of interference-dominated wireless systems including mobile, personal communications, and wireless PBX/LAN networks, we show that a significant increase in system capacity can be achieved by the use of spatial diversity (multiple antennas), and optimum combining. This is explained by the following observation: for independent flat-Rayleigh fading wireless systems with N mutually interfering users, we demonstrate that with $K+N$ antennas, $N-1$ interferers can be nulled out and $K+1$ path diversity improvement can be achieved by each of the N users. Monte Carlo evaluations show that these results also hold with frequency selective fading when optimum equalization is used at the receiver. Thus an N-fold increase in user capacity can be achieved, allowing for modular growth and improved performance by increasing the number of antennas.

1. INTRODUCTION

The chief aim of this paper is to demonstrate theoretically that antenna diversity (with optimum combining) can substantially increase the capacity of most interference-limited wireless communication systems. Increasing the number of users in a given bandwidth is the dominant goal of much of today's intense research in mobile radio, personal communication, and wireless PBX/LAN systems [1-6].

Towards this end, it is the purpose of this paper to set on sound theoretical footing some old ideas and proposals claiming that the capacity of most wireless systems can be significantly increased by exploiting the other dimension, space, that is available to the system designer. To capitalize on the spatial dimension, multiple antennas, spaced at least a half of a wavelength apart, are used to adaptively cancel the interference produced by users who are occupying the same frequency band and time slots. The interfering users can be in the same cell as the target user, and thus interference cancellation allows multiple users in the same bandwidth - in practice the number of users is limited by the number of antennas and the accuracy of the digital signal processors used at the receiver. The interferers can also be users in other cells (for frequency reuse in every cell), users in other radio systems, or even other types of radiating devices, such as microwave ovens, and thus interference cancellation also allows radio systems to operate in high interference environments.

Optimum combining and signal processing with multiple antennas, is not a new idea [2-5]. But spurred on by new theoretical results, described in the sequel, it may be one whose time has come. Use of spatial diversity is certainly made more compelling by the continued decrease in the cost of digital signal processing hardware, the advances in adaptive signal processing, and the above system benefits. Our continuous interest in this subject has recently yielded a new analytical result that is proven in the body of this paper: for a system with N users in a flat Rayleigh fading environment, optimum combining provided by a base station with $K+N$ antennas can null out $N-1$ interferers as well as achieve $K+1$ diversity improvement against multipath fading. Computer simulation shows that these results also hold with frequency selective fading when optimum equalization is used at the receiver. In addition, the average error rate, or outage probability, behaves as if each user were either spatially or frequency isolated from the other users and derives the full benefit of the shared antennas for diversity improvement. These results provide a solid basis for assessing the improvement that can be achieved by antenna diversity with optimum combining.

2. PERFORMANCE ANALYSIS

2.1 SYSTEM DESCRIPTION

Figure 1 shows a wireless system with N users, each with one antenna, communicating with a base station with M antennas. The channel transmission characteristics matrix $C(\omega)$ can be expressed as

$$C(\omega) = \left[C_1(\omega), C_2(\omega), ..., C_N(\omega)\right] \qquad (1)$$

where the N M-column vectors $C_1(\omega)$, $C_2(\omega)$,...,$C_N(\omega)$ denote the transfer characteristics from the i^{th} user,

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$i = 1,2,...,N$ to the j^{th}, $j = 1,2,...,M$ receiver or antenna. Now consider the Hermitian matrix $C^{\dagger}(\omega)C(\omega)$, where the dagger sign stands for "conjugate transpose." If the vectors in (1) are linearly independent, for each ω, then the $N \times N$ matrix inverse, $(C^{\dagger}C)^{-1}$ exists. This is a mild mathematical requirement and will most often be satisfied in practice since it is assumed that users will be spatially separated.

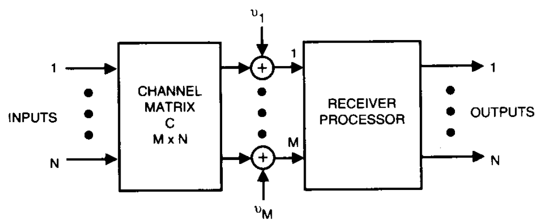

Figure 1 Multiuser communication system block diagram.

At the receiver, the M receive signals are linearly combined to generate the output signals. We are interested in the performance of this system with the optimum linear combiner, which combines the received signals to minimize the mean-square error (MSE) in the output. An explicit expression was provided for the least obtainable MSE in [7]. The formula for user "1" without loss of generality is given by

$$(MSE)_{011} = \sigma_a^2 \frac{T}{2\pi} \int_{-\pi T}^{\pi/T} \left[I + \frac{C^{\dagger}(\omega)C(\omega)}{N_0} \sigma_a^2 \right]_{11}^{-1} d\omega \quad (2)$$

where $\sigma_a^2 = E \mid a_n^{(1)} \mid^2$, $[\ ]_{11}^{-1}$ stands for the "1 1" component of a matrix, T is the symbol duration, N_0 is the noise density, and $a_n^{(1)}$ are the 1^{st} user's complex data symbols.

2.2 FLAT RAYLEIGH FADING

With flat Rayleigh fading, the channel matrix $C(\omega)$ is independent of frequency and all the elements of C can be regarded as independent, zero-mean, complex Gaussian random variables with variance σ_i^2 for the i^{th} user, provided the antenna elements are separated by at least half a wavelength. Let us consider the high signal-to-noise case (which also results in the zero forcing optimum combiner solution). Under these assumptions (2) reduces to

$$(MSE)_{011} = (C^{\dagger}C)_{11}^{-1} N_0 \ . \quad (3)$$

It can be shown that the MSE for any signal-to-noise is upper bounded by (3) and therefore the zero-forcing solution serves as an upper bound on the MSE solution. For these reasons and the fact that it is easier to analyze the zero-forcing structure, we proceed in this paper with this approach. Using (3), we find that an exponentially tight upper bound on the conditional probability of error is given by

$$P_{e1}(C) \leq \exp\left\{ -\frac{\rho}{\sigma_a^2} \frac{1}{(C^{\dagger}C)_{11}^{-1}} \right\}, \quad (4)$$

where ρ is the signal-to-noise ratio for user "1", i.e., $\rho = \frac{\sigma_a^2 \sigma_1^2}{N_0}$.

In order to analyze the performance of the general set-up, we must be able to determine the statistical properties of the random variable $\alpha = 1/(C^{\dagger}C)_{11}^{-1}$. From the definition of the inverse of a matrix we express this quantity as follows,

$$\alpha = \frac{\det (C^{\dagger}C)}{A_{11}} = \frac{\Delta_N(C_1,...,C_N)}{\Delta_{N-1}(C_2,...,C_N)} \quad (5)$$

where $\det(\cdot)$ stands for determinant, A_{11} is the "11" cofactor, $\Delta_N(C_1,...,C_N) = \det(C^{\dagger}C)$, and $\Delta_{N-1}(C_2,...,C_N)$ is the determinant resulting from striking out the first row and first column of $C^{\dagger}C$. From the definition of the determinant

$$\Delta_N(C_1,C_2,...,C_N) = \sum \pm C_1^{\dagger}C_{i_1} C_2^{\dagger}C_{i_2} \cdots C_N^{\dagger}C_{i_N} \ , \quad (6)$$

where the sum is extended over all $N!$ permutations of $1,2,..., N$, the "+" sign is assigned for an even permutation and "-" for an odd permutation, it can be seen that it is possible to factor out $C_1^{\dagger}$ on the left and C_1 on the right in each term. This factorization makes it possible to express Δ_N in the following form

$$\Delta_N(C_1,C_2,...,C_N) = C_1^{\dagger} F(C_2,C_3,...,C_N) C_1 \quad (7)$$

where F is an $M \times M$ matrix independent of C_1. By normalizing F by $\Delta_{N-1}(C_2...C_N)$ so that $F/\Delta_{N-1} = \tilde{M}$, we can express the quantity of interest as a positive quadratic form

$$\alpha = C_1^{\dagger} \tilde{M} C_1 \quad (8)$$

where $\tilde{M}$ is Hermitian and non-negative. Diagonalizing $\tilde{M}$ by a unitary transformation ϕ, we write for α

$$\alpha = C_1^{\dagger} \phi^{\dagger} \Lambda \phi C_1 = z^{\dagger} \Lambda z$$

$$\alpha = \sum_{i=1}^{M} \lambda_i \mid z_i \mid^2 \qquad (9)$$

where Λ is $diag(\lambda_1 \cdots \lambda_M)$, λ_i's being the eigenvalues of $\tilde{M}$, $z = \phi\, C_1$, and $z_i = (\phi C_1)_i$, $i = 1, ..., M$.

Since C_1 is a complex Gaussian vector, so is z conditioned on ϕ. Also, the vectors C_1 and z possess identical statistics since ϕ is unitary. Therefore, conditioned on the eigenvalues, the random variable α is a sum-of-squares of Gaussian random variables and therefore has a known probability distribution.

One would expect the actual distribution of α to be rather complicated since for example the characteristic function of α, conditioned on the eigenvalues, is readily evaluated in the form

$$E\left\{ e^{i\alpha\omega} \middle| \lambda_i,\, i=1,...,M \right\} = \prod_{i=1}^{M} (1 - 2\omega\lambda_i)^{-1} \ . \quad (10)$$

But since the eigenvalues are complicated nonlinear functions of the remaining $N-1$ vectors, $(C_2, C_3, ..., C_N)$, the actual characteristic function of α, the average of (10) with respect to the eigenvalues, appears to be intractable. A remarkable discovery, totally unexpected, revealed that the eigenvalues of $\tilde{M}$ are equal to either 1 or zero, with $M-N+1$ eigenvalues equal to 1. This astonishing fact makes it possible to claim that α is Chi-square distributed.

Applying this result in (4), we evaluate explicitly the average probability of error, i.e.,

$$P_e = E_C\, P_e(C) \le E_C \exp\left\{ -\frac{\rho}{\sigma_a^2} \frac{1}{(C^\dagger C)_{11}^{-1}} \right\} = E_\alpha e^{-\frac{\rho}{\sigma_a^2}\alpha}$$

$$= E_z \exp\left\{ -\frac{\rho}{\sigma_a^2} \sum_{i=1}^{M-N+1} \mid z_i \mid^2 \right\} = \left[1 + \frac{\rho}{\sigma_a^2} \right]^{-(M-N+1)} \ . \quad (11)$$

Thus, the average probability of error with optimum combining, M antennas, and N interferers is the same as maximal ratio combining with $M-N+1$ antennas and no interferers.

The physical implications of this result are as follows. The error rate of a particular user is unaffected by all other users. It only depends on the user's own SNR, ρ. Of course, the price paid is in the diminished diversity benefits obtained for each user. For, when the number of antennas M equals the number of users N, the average error rate is as if there was only one antenna per user. But remarkably, the resulting performance is as if all the other users or interferers did not exist. The nulling-out of other users results only in reduced diversity benefits. But even when $M=N+1$, all users enjoy dual diversity, i.e., the addition of each antenna adds diversity to every user.

2.3 FREQUENCY SELECTIVE FADING

With frequency selective fading, unfortunately, no closed form analytical results exist as for the flat fading case. The problem is complicated since in this case the variances of the output noise samples are complicated functionals of the matrix channel characteristics, $C(\omega)$. From (2) [see also (4)], an exponentially tight bound on the conditional probability of error is given by

$$P_e(C(\omega)) \le \exp\left\{ -\frac{\rho}{\sigma_a^2} \frac{1}{\sigma^2(C)} \right\} \qquad (12)$$

where

$$\sigma^2(C(\omega)) = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left[C^\dagger(\omega) C(\omega) \right]_{11}^{-1} d\omega \ . \qquad (13)$$

The outage probability as well as the average probability of error depends in a complicated way on the statistical characterization of the matrix $C(\omega)$.

If we assume that the propagation mode is by uniformly distributed scatterers and delay spread cannot be neglected, then a reasonable statistical model for $C(\omega)$ is the following. For each frequency ω, every entry in $C(\omega)$ is complex Gaussian, but at different frequencies the entries are correlated. Specifying the multidimensional correlation function provides a complete statistical characterization of the matrix medium. For this model, which is often referred to as the "selective fading" Rayleigh medium, we can derive an upper bound on the average probability of error. Also, for a two ray model of the frequency selective Rayleigh process for each entry of the matrix C, we have carried out Monte Carlo evaluations. We will discuss these results later, but first we provide an outline of our bounding technique.

Note that from the properties of the matrix $C^\dagger(\omega) C(\omega)$, irrespective of the statistics, we can always express the noise variance for any frequency ω as

$$\sigma^2(C(\omega)) = \left\langle \frac{1}{\sum_{i=1}^{M-N+1} \mid z_i(\omega) \mid^2} \right\rangle_\omega \qquad (14)$$

where $\langle\cdot\rangle_\omega = \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left[C^\dagger(\omega) C(\omega) \right]_{11}^{-1} d\omega$ and $z_i(\omega) = \phi_i^\dagger(\omega) C_1(\omega)$ where $\phi_i(\omega)$ are the eigenvectors of the matrix M. We now note that for each frequency

$$\alpha(\omega) = \sum_{i=1}^{M-N+1} | z_i(\omega) |^2 \qquad (15)$$

is Gamma distributed with probability density

$$p(\alpha) = \frac{\alpha^{K-1} e^{-\alpha}}{(K-1)!} \qquad (16)$$

where $K = M - N + 1$.

Making use of these facts, it can be shown that an upper bound on the average probability of error is

$$\overline{P_e} = E_C P_e(C(\omega)) \le d_{M-N} \left[\frac{\sigma_a^2}{\rho} \right]^{M-N} \qquad (17)$$

where $d_{M-N} = \frac{1 \cdot 3 \cdot 5 \cdots [2(M-N)-1]}{(M-N)!}$. While this may appear to be a loose upper bound, it does indicate that when the number of antenna elements is not much greater than the number of users or interferers we only lose the diversity benefit from one additional antenna.

As an illustration, suppose that $M-N=1$, i.e., one more antenna element than users. Our bound indicates that $\overline{P_e} \le 1/\rho$ for a binary system when $\sigma_a^2 = 1$. On the other hand, when only flat fading is present, we can expect $\overline{P_e} \le \frac{1}{\rho^2}$.

In actual Monte Carlo evaluation of averages presented below, we found that the average error rates were much lower than predicted from (17).

Before proceeding, we note that with a two ray model of frequency selective fading with $N=1$ (no interference), [2] provides bounds showing that the average bit error rate decreases with increasing time delay between the two multipath rays when optimum combining and equalization is used. For this two ray model,

$$c_{ij} = a_{ij} + b_{ij} e^{-j\omega\tau_i} \qquad (18)$$

where a_{ij} and b_{ij} are complex Gaussian random variables with zero mean and variance 1/2, and τ_i is the time delay between the two rays.

To gain insight into the behavior of average error rate versus delay spread, we used Monte Carlo simulation to derive 1000 channel matrices C and numerically calculated the average bit error rate for each channel from (12). The entries in C are given in (18). The bit error rate averaged over these 1000 C matrices is shown in Figures 2 and 3.

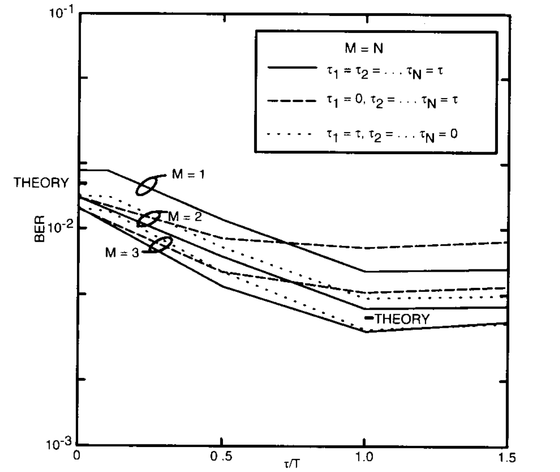

Figure 2 Effect of frequency selective fading for $M=N$.

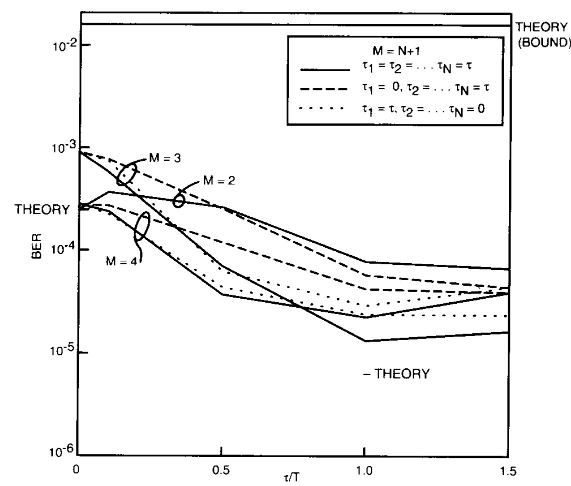

Figure 3 Effect of frequency selective fading for $M=N+1$.

Figure 2 shows the average bit error rate versus τ/T, where T is the symbol duration, for $M=N$ with a) frequency selective fading of the desired and interfering signals, $\tau_1 = \tau_2 = \cdots = \tau_N = \tau$, where τ_1 is the time delay between the two multipath rays of the desired signal and $\tau_2, \cdots, \tau_N$ is the time delay of the interfering signals, b) frequency selective fading of the desired signal only, $\tau_1 = \tau$, $\tau_2 = \cdots = \tau_N = 0$, and

c) frequency selective fading of the interferers only, $\tau_1=0$, $\tau_2=\cdots=\tau_N=\tau$. Results for $M = 1$, 2, and 3 are in good agreement with the theory for flat Rayleigh fading (11) and for frequency selective fading with $M = 1$ [2]. Figure 3 shows the average bit error rate versus τ/T for $M=N+1$. Again, the results are in good agreement with (11) for $\tau=0$ and with [2] for $\tau/T=1$.

3. EXPERIMENTAL RESULTS

Let us now describe an experiment that verifies some of these concepts. To demonstrate and test the interference nulling ability of optimum combining in a fading environment, an experimental system was built. Figure 4 shows a block diagram of the experiment, which consisted of 3 users (remotes), a 24 channel Rayleigh fading simulator, 8 receive antennas, and a DSP32C processor at the receiver. The three remotes' signals used QPSK modulation, at a common 50 MHz IF frequency, consisting of a biphase data signal and a quadrature biphase signal with a pseudorandom code that was unique to each user. This pseudorandom code was used to generate a reference signal at the receiver to distinguish the remotes. The fading simulator generated the 8 output signals for the antennas by combining the three remotes' signals with independent flat, Rayleigh fading between each input and antenna output. The fading rate of the simulator was adjustable up to 81 Hz. The outputs of the simulator were demodulated by the 8 antenna subsystems, A/D converted, multiplexed, and input to a DSP32C. This DSP32C used the LMS algorithm to acquire and track one of the remote's signals. With our program in the DSP32C, the maximum weight update rate was 2 kHz, and the data rate was set to 2 kbps for convenience (although any data rate greater than 2 kbps could have been used). The experiment successfully demonstrated the suppression of 2 interferers for a 3-fold capacity increase even with a fading rate of 81 Hz. Note that this corresponds to a data rate to fading rate ratio of 25, which is much lower than that required in most systems. In all cases the bit error rate did not exceed 10^{-2}. Noise on the circuitry backplane limited the accuracy of the A/D to 6 bits, which did not allow verification of the 6-fold diversity improvement predicted by (11).

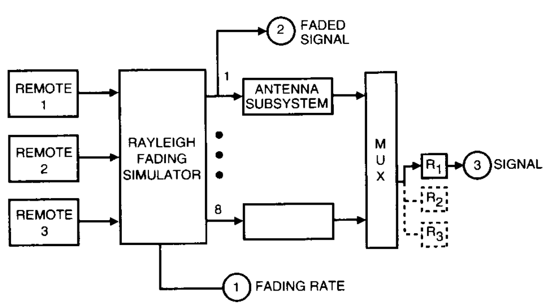

Figure 4 Experimental system.

4. SUMMARY

For a broad class of interference-dominated wireless systems including mobile, personal communications, and wireless PBX/LAN networks, we have shown that a significant increase in system capacity can be achieved by the use of spatial diversity (multiple antennas) and optimum combining. This increase in user capacity may be achieved with a modest increase in complexity. Moreover, the system naturally lends itself to modular growth and improved performance by increasing the number of antennas.

5. ACKNOWLEDGEMENTS

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