Effect of Fading Correlation on Adaptive Arrays in Digital Mobile Radio

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***Abstract*—In this paper, we investigate the effect of correlations among the fading signals at the antenna elements of an adaptive array in a digital wireless communication system. With an adaptive array, the signals received by multiple antennas are optimally weighted and combined to suppress interference and combat desired signal fading. Previous results for flat and frequency-selective fading assumed independent fading at each antenna. Here, we present a model of local scattering around a mobile where the received multipath signals arrive at the base station within a given beamwidth, and derive a closed-form expression for the correlation as a function of antenna spacing. Results show that the degradation in performance with correlation in an adaptive array that combats fading and suppresses interference is only slightly larger than that for combating fading alone, i.e., with maximal ratio combining. This degradation is small even with correlation as high as 0.5.**

I. Introduction

Antenna arrays with optimum combining combat multipath fading of the desired signal and suppress interfering signals, thereby increasing both the performance and capacity of wireless systems. This increase is reduced, however, by correlation of the fading signals between the received antennas.

Previous theoretical and computer simulation studies of optimum combining (e.g., [1]–[6]) assumed independent fading of the desired and interfering signals at each receive antenna. Such independence occurs if multipath reflections are uniformly distributed around receive antennas that are spaced at least a half wavelength apart. However, the signals often arrive at the receive antennas mainly from a given direction. For example, in rural or suburban mobile radio, a high base station antenna typically has a line-of-sight to within the vicinity of the mobile, with local scattering around the mobile generating signals that arrive mainly within a given range of angles or beamwidth. This problem was studied in [7], where theoretical and experimental results showed the relationship of angle of arrival and beamwidth with the correlation of fading between antennas. Specifically, as the angle of arrival approaches end-fire (parallel to the array) and the beamwidth decreases, the antenna spacing must be increased to reduce correlation. When this correlation is high (>0.8), because the signals at the antennas tend to fade at the same time, the diversity benefit of antenna arrays against fading (i.e., with maximal ratio combining) is significantly reduced [8]. On the other hand, because independent fading is not required for interference suppression, antenna arrays can suppress interference even with complete correlation (=1), i.e., in line-of-sight systems without multipath. In particular, theoretical and computer simulation results [1], [3], [4], [9], [10] have shown that with M antennas, $M-1$ interferers can be completely suppressed in both fading (with zero correlation) and nonfading (with complete correlation) environments. Thus, we need to understand the antenna array performance with joint fading reduction and interference suppression. In addition, the effect of correlation with frequency-selective fading, when equalization is also used, must be evaluated.

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This paper considers the effect of correlation of the signal fading at the antennas of an adaptive array with optimum combining to combat desired signal fading and suppress interference, and optimum linear equalization to combat frequency-selective fading. We first present a model of local scattering where the received multipath signals arrive within a given beamwidth. We derive a closed-form expression for the fading correlation with this model as a function of the angle of arrival, beamwidth, and antenna spacing. Using these theoretical results with Monte Carlo simulation, we then generate results for the effect of beamwidth (i.e., correlation) on the adaptive array performance with given antenna spacing and random angles of arrival. Results are presented for optimum combining with flat fading, as well as for frequency-selective fading, using a two-path delay spread model with joint optimum combining and linear equalization. Computer simulation results show that the degradation in performance with correlation in an adaptive array that combats fading and suppresses interference is slightly larger than that for combating fading alone, i.e., with maximal ratio combining. This degradation is small even with correlation as high as 0.5. Results for an adaptive array with either flat Rayleigh fading or frequency-selective fading show that with an antenna spacing of four wavelengths, there is little performance degradation as long as the beamwidth of the received signals is greater than 20°. Further increases in antenna spacing would reduce this beamwidth even more.

In Section II, we describe optimum combining and equalization with antenna arrays and discuss how fading correlation can occur. The model and theoretical analysis of wireless systems with fading correlation is presented in Section III. In Section IV, we describe the computer simulation technique and present results on the performance degradation with correlation. A summary and conclusions are presented in Section V.

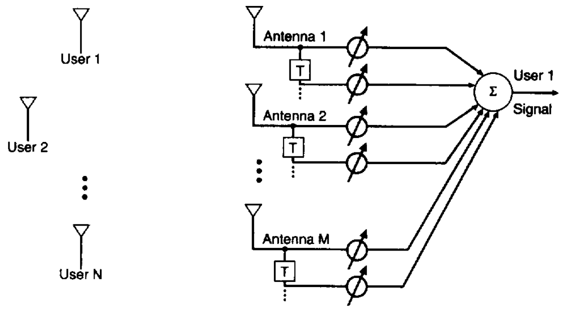

Fig. 1. Wireless communication system employing adaptive arrays, where a base with M antennas receives signals from N users.

II. BACKGROUND

Fig. 1 shows a digital wireless communication system employing adaptive arrays where a base with M antennas receives signals from N users. These N users operate in the same bandwidth simultaneously and include signals destined to the base, as well as those destined to other bases, but interfering with the desired signals, as in cellular systems.

Let the complex channel transfer function from user "i" to antenna "j" be denoted as $c_{ij}(\omega)$. Then, the channel vector from user "i" to the base antennas is $C_i(\omega) = [c_{i1}(\omega)\ldots c_{iM}(\omega)]^T$, where the superscript T denotes transpose, and the $M \times N$ channel matrix between the N users and the base is given by

$$C(\omega) = [C_1(\omega)\ldots C_N(\omega)]. \quad (1)$$

In this paper, we are interested in linear processing at the base station of the M received signals to generate an output signal that corresponds to the data from one desired signal (user 1). Specifically, we consider ideal optimum combining and linear equalization, where the M received signals are combined to minimize the mean-square error (MSE) in the output. For ideal optimum combining, we assume perfect tracking of the desired and interfering signals by the combining algorithm at the base station. For ideal linear equalization, we consider a synchronous tapped delay line with an infinite number of taps, as shown in Fig. 1. This equalizer is the optimum linear equalizer under the assumption that the desired signal spectrum is bandwidth-limited to the data rate ($1/T$). As shown before [1], with ideal optimum combining and linear equalization, the minimum MSE for user "1" for given $C(\omega)$ is given by

$$\mathrm{MSE}[C] = \sigma_a^2 \frac{T}{2\pi} \int_{-\pi/T}^{\pi/T} \left[I + \rho C^\dagger(\omega) C(\omega)\right]_{11}^{-1} d\omega, \quad (2)$$

where I is an $N \times N$ identity matrix, ρ is the signal-to-noise ratio, and $\sigma_a^2 = E\left[|a_n|^2\right]$, where the a_n's are the complex data symbols. The superscript denotes complex conjugate transpose, and $[\ ]_{11}^{-1}$ stands for the "11" component of the inverse of a matrix. For coherent detection of binary phase shift keyed (BPSK) or quadrature amplitude modulated (QAM) signals, the error rate can then be upper bounded by

$$P_e \leq E_C\left[\exp\left[-\frac{1-\mathrm{MSE}[C]/\sigma_a^2}{\mathrm{MSE}[C]}\right]\right], \quad (3)$$

where $E_C[\ ]$ is the expected value with respect to the channel matrices.

With multipath, the $c_{ij}(\omega)$'s are modeled as complex Gaussian random variables at each frequency ω. The variation of $c_{ij}(\omega)$ with ω depends on the delay spread model of the channel. In this paper, we examine numerically two such models: 1) flat fading, i.e., $c_{ij}(\omega) = c_{ij}$ for all ω, where equalization is not needed, and 2) a two-path delay spread model,

$$c_{ij}(\omega) = c_{ij}^1 + c_{ij}^2 e^{-j\omega\tau_i}, \quad (4)$$

where τ_i is the time delay between the two paths for the i-th user and c_{ij}^1 and c_{ij}^2 are complex Gaussian random variables, and the fading in the two paths with different time delays is independent, i.e., c_{ij}^1 is independent of c_{ij}^2 (but the c_{ij}'s are not necessarily independent).

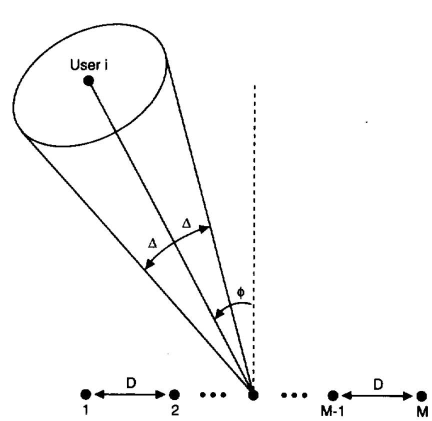

Fig. 2. Wireless environment where all signals from a mobile arrive at the base station within $\pm\Delta$ of angle ϕ.

Previous papers have assumed that the c_{ij}'s are independent. Such independence occurs if multipath reflections are uniformly distributed around the receive antennas that are spaced at least a half wavelength apart (this situation is examined in detail below). However, the signals often arrive at the receive antennas mainly from a given direction. For example, in rural or suburban mobile radio, a high base station antenna typically has a line-of-sight to within the vicinity of the mobile, with local scattering around the mobile generating signals that arrive mainly within a given range of angles or beamwidth. Fig. 2 shows a typical scenario where all signals from a mobile arrive at the base station within $\pm\Delta$ at angle ϕ. This problem was studied in [7], where theoretical and experimental results showed the relationship of angle of arrival and beamwidth with the correlation of fading between antennas. Specifically, [7] assumed that the probability density function for the angle of arrival of the i-th ray is given by

$$p(\phi_i) = \frac{Q}{\pi}\cos^n(\phi_i - \phi) \qquad -\frac{\pi}{2} + \phi \le \phi_i \le \frac{\pi}{2} + \phi \quad (5)$$

where n is an even integer chosen to determine the beamwidth and Q is a normalizing constant chosen to make $p(\phi_i)$ a density function. The correlation of the fading between two antennas spaced D apart is then [7]

$$R_{xx} = \int_{-\pi/2+\phi}^{\pi/2+\phi} \cos(2\pi D/\lambda \sin(\phi_i - \phi))p(\phi_i)d\phi_i \quad (6)$$

and

$$R_{xy} = \int_{-\pi/2+\phi}^{\pi/2+\phi} \sin(2\pi D/\lambda \sin(\phi_i - \phi))p(\phi_i)d\phi_i \quad (7)$$

where $\lambda = \omega/(2\pi c)$, c is the speed of light, R_{xx} is the correlation between the real parts of c_{ij} and c_{ik}, and R_{xy} is the correlation between the real part of c_{ij} and the imaginary part of c_{ik}. Unfortunately, (6) and (7) must be evaluated numerically. Therefore, in the next section, we present a generic model, where the probability density function of ϕ_i is assumed to be uniform,

$$p(\phi_i) = \begin{cases} \frac{1}{2\Delta} & -\Delta + \phi \le \phi_i \le \Delta + \phi \\ 0 & \text{elsewhere} \end{cases}. \quad (8)$$

This allows for the derivation of a closed-form expression for the correlation coefficient, with results that agree with the results obtained numerically using the model of [7] (see Section IV).

III. Channel Model

We develop a mathematical model for multipath media applicable in wireless digital communications employing antenna array processors. The model is useful for the evaluation of signal correlations among the antenna array elements which are critically important in determining ultimate system performance. The degree of correlation depends on the element spacings and signal scattering angles resulting from the physical surroundings.

Using the model of a uniform probability density function of ϕ_i (8), in the Appendix we derive closed-form expressions for the correlation of the fading between the j-th and k-th antennas (as compared to (6) and (7)).

Note that using these expressions, (A-19) and (A-20), when $\Delta = \pi$,

$$\tilde{R}_{xx}(k-j) = \tilde{R}_{yy}(k-j) = J_o(z(k-j))$$

and

$$\tilde{R}_{xy}(k-j) = \tilde{R}_{yx}(k-j) = 0,$$

where $z = 2\pi\frac{D}{\lambda}$. The implication of these results is that when reflections are allowed to arrive at the antenna array from all directions, the correlation of signals at adjacent antenna array elements is determined from $J_0(z) = 0$ which implies that $z = 2\pi\frac{D}{\lambda} \approx 2.4$ or $\frac{D}{\lambda} \approx \frac{2.4}{2\pi} \approx .382$. This sets the minimum spacings between antenna elements yielding zero correlation.

The overall channel matrix C can now be expressed in terms of the M-column vectors (see (1)),

$$C(\omega) = [C_1(\omega)C_2(\omega)...C_N(\omega)] \quad (9)$$

where the column vector $C_k(\omega)$ represents the transmission characteristics from user "k" to all the antenna elements. Since each user is characterized by its own surroundings, and if the users are not on top of one another to within wavelengths, it is reasonable to assume that the columns in (9) are statistically independent. Consequently, we need only to characterize the correlation properties of a typical user, which we have already accomplished.

Expressing the complex column vector $C_k(\omega) = \boldsymbol{x}_k(\omega) + i\boldsymbol{y}_k(\omega)$ where $\boldsymbol{x}_k$ and $\boldsymbol{y}_k$ are the real and imaginary M-column vectors associated with user "k," we define the $2M$ augmented column vector as

$$\Lambda_k = \begin{pmatrix} x_{k1} \\ y_{k1} \\ x_{k2} \\ y_{k2} \\ \cdot \\ \cdot \\ x_{kM} \\ y_{kM} \end{pmatrix} \quad (10)$$

and seek to evaluate the $2M \times 2M$ correlation matrix

$$\hat{R}_k = E\left[\Lambda_k\Lambda_k^\dagger\right], k = 1, ...N. \quad (11)$$

Defining the 2×2 matrix

$$D_{|i-j|} = \begin{bmatrix} \tilde{R}_{xx}(i-j) & \tilde{R}_{xy}(|i-j|) \\ -\tilde{R}_{xy}(|i-j|) & \tilde{R}_{yy}(i-j) \end{bmatrix} \quad i, j = 1, ..., M \quad (12)$$

where the entries are given in (A-19) and (A-20), it is easy to see that $\hat{R}_k$ can be represented in terms of these block 2×2 matrices as follows:

$$\frac{\hat{R}_k}{\sigma_k^2} = \begin{bmatrix} I_{2\times2} & D_1 & D_2 & \cdot & D_M \\ D_1^T & I_{2\times2} & D_1 & \cdot & D_{M-1} \\ D_2^T & D_1^T & I_{2\times2} & \cdot & D_{M-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ D_M^T & D_{M-1}^T & D_{M-2}^T & \cdot & I_{2\times2} \end{bmatrix} \quad (13)$$

where σ_k^2 is the received signal power for the k-th user (see (A-16), with a subscript "k" denoting the k^{th} user—(A-16) applies to a typical user). Clearly, $\hat{R}_k$ is a Toeplitz matrix.

IV. Results

A. Correlation

Let us first consider the correlation as a function of the antenna spacing D/λ, angle of arrival ϕ, and beamwidth Δ. When the signal arrives from broadside ($\phi = 0°$), $R_{xy} = 0$ for all D/λ. Thus, the envelope correlation, $R = \left(|R_{xx}|^2 + |R_{xy}|^2\right)^{1/2}$ is just $|R_{xx}|$. R_{xx} versus antenna spacing is shown in Fig. 3 for $\Delta = 180°, 90°, 40°, 20°, 10°$, and $3°$. These results agree with results using the model of [7] with Δ equivalent to the 3 dB beamwidth of [7]. The figure shows that, as Δ decreases, the

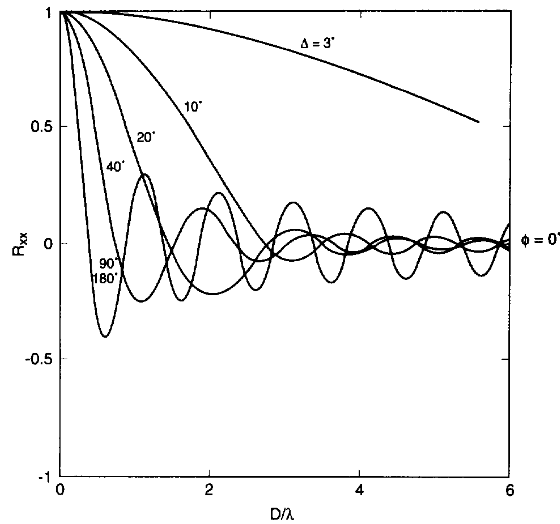

Fig. 3. Correlation of the real portion of the fading versus antenna spacing for $\phi = 0°$.

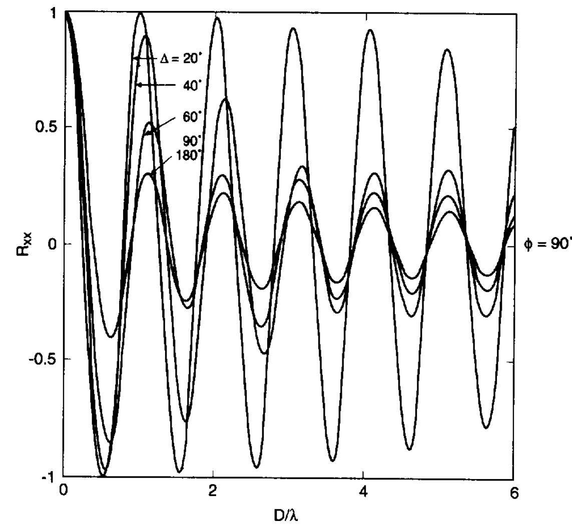

Fig. 4. Correlation of the real portion of the fading versus antenna spacing for $\phi = 90°$.

first zero in the correlation occurs at larger antenna spacing. Specifically, the first zero-crossing occurs at $D/\lambda \approx 30/\Delta$, with Δ in degrees. Thus, these results depend mainly on $D/\lambda/\Delta$, and show that independent fading occurs when the antenna beamwidth from two elements of the array is about the same as the beamwidth of the arriving signal.

When the signal arrives from other than broadside, $\phi \neq 0°$, the antenna spacing for low correlation increases and the envelope correlation is never zero for almost all values of $\phi \neq 0°$ and $\Delta < 180°$ (since zero envelope correlation requires that R_{xx} (A-27) and R_{xy} (A-28) have zero crossings at exactly the same spacing). Fig. 4 shows R_{xx} versus antenna spacing for the worst case of $\phi = 90°$. For R_{xy}, the peak value of the oscillations is similar to that shown in Fig. 4. Note that the correlation decreases much more slowly with antenna spacing at $\phi = 90°$ than at $\phi = 0°$.

Fig. 5 shows the antenna spacing required for the envelope correlation to remain below 0.5 as a function of ϕ and Δ. The required spacing is only a few wavelengths up to very small beamwidths, unless ϕ is close to 90°.

Experimental measurements of the beamwidth in mobile radio are presented and discussed in [7], [12]. These results show that, as expected, the beamwidth decreases with the antenna height. Fortunately, in most cases, antenna spacings on the order of only 10λ (several feet at 900 MHz) are required to obtain low correlation.

The effect of correlation on reducing the effectiveness of antenna diversity against desired signal fading is shown in [8]. With maximal ratio combining and two antennas, small correlation (< 0.3) has a negligible effect on performance, and the degradation is small unless the correlation is large (> 0.8).

The effect of correlation on reducing the effectiveness of antenna arrays against interference suppression is as follows. With M antenna elements, the array has $M - 1$ degrees of freedom. Thus, as shown by theoretical and computer simulation results [1], [3], [4], [9], [10], an M antenna element array can null out $M - 1$ interfering signals independent of the fading correlation (i.e., with or without fading). The only factor that changes with the environment is the required spatial separation of the interfering signals: without fading, the signals must be separated from the desired signal by the antenna beamwidth, while with fading (with $\Delta = 180°$), the signals need only be separated by about half a wavelength. For $\Delta < 180°$, we note the following. Spacing the receive antennas at greater than $\lambda/2$ decreases the beamwidth of the array but also creates grating lobes, i.e., the antenna pattern repeats every $90°/(D/\lambda)$. Because of these grating lobes, with large antenna spacing to reduce fading correlation, interfering

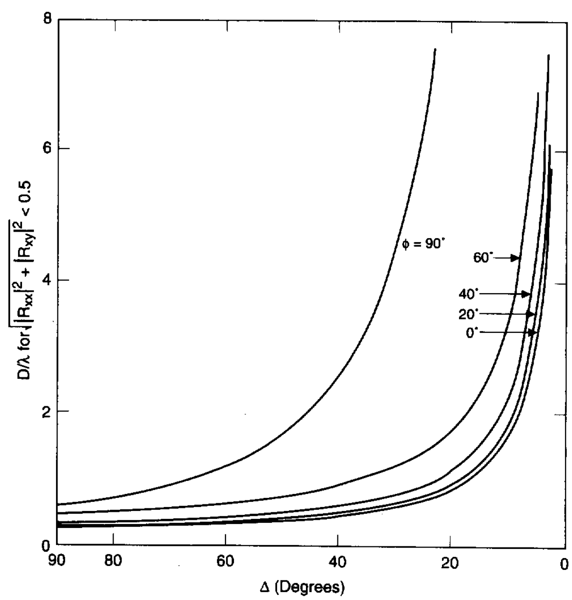

Fig. 5. Antenna spacing required for the envelope correlation to remain below 0.5 as a function of ϕ and Δ.

signals outside of the antenna beamwidth can not always be suppressed. However, because of the multipath fading, most interfering signals within the beamwidth but separated by at least half a wavelength, can be suppressed. Therefore, as before, only the required spatial separation of the interfering signals changes with the environment. Thus, as the signal beamwidth decreases (i.e., as the correlation increases), the effectiveness of adaptive arrays to suppress interference alone does not change, but the effectiveness against fading does.

B. Performance with Fading and Interference

The effect of correlation on an adaptive array that jointly suppresses interference and reduces fading effects was determined in the following manner. For fixed D/λ and the same fixed Δ for the desired and interfering signals, we use Monte Carlo simulation to derive 10,000 channel matrices C with random ϕ and fading and then calculate the performance averaged over these matrices (i.e., ϕ and fading). We assume that the users are randomly located (separated by at least half a wavelength) and thus ϕ is an independent random variable for each user with a uniform probability density function, i.e.,

$$p(\phi) = \begin{cases} \frac{1}{2\pi} & -\pi < \phi \leq \pi \\ 0 & \text{elsewhere.} \end{cases} \tag{14}$$

The performance measures we consider are the average error rate, as well as the outage probability, i.e., the probability that the error rate exceeds a given value.

The error rate for a given ϕ and fading was calculated as follows. For given ϕ for each user, Δ, and D/λ, the correlation matrix $\hat{R}_k/\sigma_k^2$ for each user was calculated using (13). To generate C, we first generate a $2M$ vector A_k for each user with each element a_{ki} being an independent, zero-mean Gaussian random variable with a variance of 1/2. Thus,

$$A_k = [a_{k1} \ldots a_{kM}]^T \tag{15}$$

for the k-th user. The k-th column of C, C_k, is then

$$C_k = \frac{\hat{R}_k^{1/2}}{\sigma_k^2} A_k. \tag{16}$$

Note that $\frac{\hat{R}_k^{1/2}}{\sigma_k^2}$ is given by

$$\frac{\hat{R}_k^{1/2}}{\sigma_k^2} = x \begin{bmatrix} \sqrt{\lambda_1} & 0 & \cdot & 0 \\ 0 & \sqrt{\lambda_2} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \sqrt{\lambda_{2M}} \end{bmatrix} x^T, \tag{17}$$

where $x = [x_1 \ldots x_{2M}]^T$, and x_i and λ_i are the eigenvectors and eigenvalues of $\frac{\hat{R}_k}{\sigma_k^2}$, respectively. For frequency-selective fading with two-path delay-spread (4), the above procedure was repeated twice to obtain the c_{ij}^1's and c_{ij}^2's. The MSE is then given by (2) and the error rate by (3).

We first consider the effect of correlation with flat fading, two receive antennas, and one interferer with the same power as the desired signal. Fig. 6 exhibits the average error rate versus Δ with $\rho = 18$ dB and 27 dB, and $D = 0.382\lambda$ and 3.82λ. Note that $D = 0.382\lambda$ corresponds to zero correlation when the signal arrives uniformly from all angles

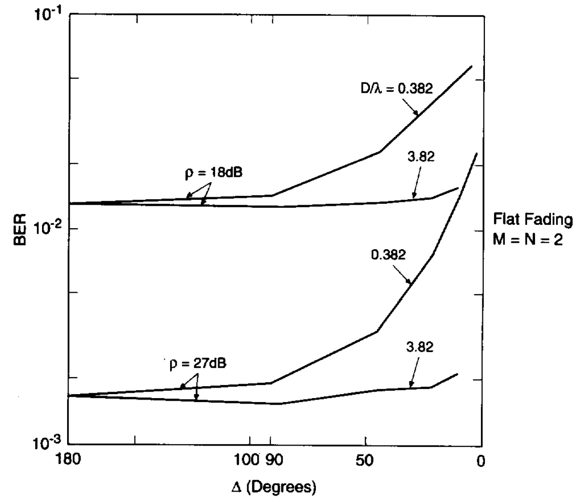

Fig. 6. Average error rate versus Δ with flat fading and $M = N = 2$.

($\Delta = 180°$). At $D = 0.382\lambda$, the performance is degraded slightly at $\Delta = 90°$ and becomes much worse with smaller Δ. However, at $D = 3.82\lambda$, there is little degradation until Δ is 10°–20°. Thus, increasing the antenna spacing by a factor of 10 decreases the tolerable Δ by about a factor of 10 as well (corresponding to the decrease in antenna beamwidth as discussed in Section IV-A). As shown in Fig. 5, at 20°, the correlation is about 0.5 in the worst case of $\phi = 90°$. Fig. 6 also shows that the degradation with Δ is larger with higher ρ, but the above conclusions are the same. Similar results were obtained for the outage probability.

Fig. 7 shows the outage probability versus Δ with flat fading, $N - 1$ equal-power interferers, and $M = N + 1$. Results are shown for the probability of exceeding a 10^{-2} error rate, with $\rho = 17$ dB, and $D = 0.382\lambda$ and 3.82λ as in Fig. 6. As compared to Fig. 6, these results show that for $D = 0.382\lambda$ correlation degrades the performance more when there is an additional antenna. Additional results we obtained for $M = N + 2$ and $M = N + 3$ show that the degradation with correlation grows even larger with more antennas. In Fig. 7, the $M = 2$ results are without interference and thus correspond to the performance with maximal ratio combining. The performance with $D = 0.382\lambda$ is seen to be degraded somewhat more by correlation when interference must also be suppressed (i.e., $M = 3$ and 4 versus $M = 2$ results). However, in all cases when the spacing is increased to $D = 3.82\lambda$, the performance remains constant[1] as long as Δ is greater than about 20°, i.e., the correlation is below 0.5.

Finally, we consider the effect of correlation with frequency-selective fading when joint optimum combining and equalization is used. Fig. 8 shows the average error rate versus Δ with two-path delay spread and $M = N = 2$. Results are for $\rho = 17$ dB, $D = 0.382\lambda$ and 3.82λ, and delay $\tau = 0$, 0.7 T, and T for the desired and interfering signals. Note that

[1] The outage probability is seen in Fig. 7 to increase slightly with increasing Δ at one point for both $M = 3$ and 4 with $D/\lambda = 3.82$, but this is just a numerical aberration due to using only 10,000 channel matrices in the simulations.

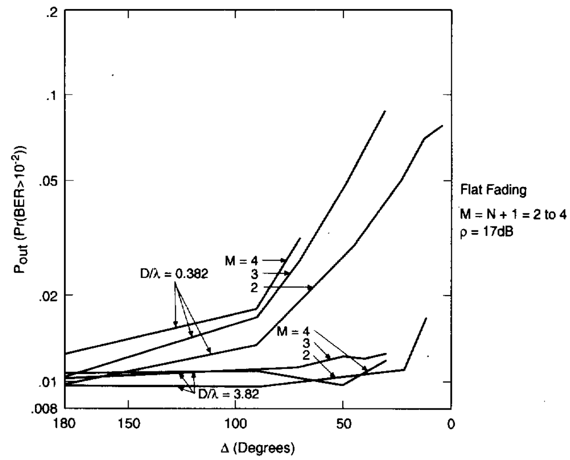

Fig. 7. Outage probability versus Δ with flat fading and $M = N + 1$.

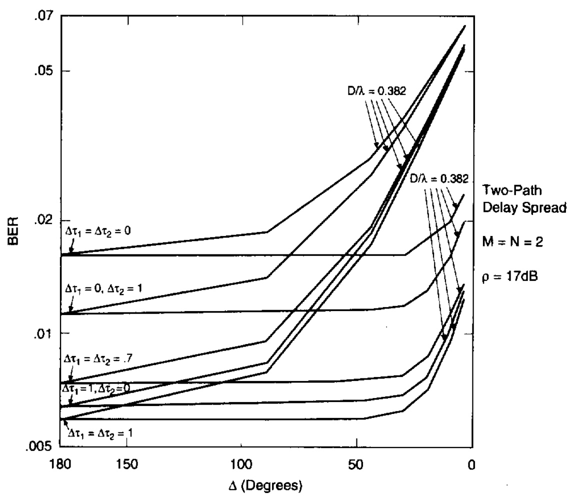

Fig. 8. Average error rate versus Δ with two-path delay spread and $M = N = 2$.

the error rate decreases with τ, due to the diversity benefit of frequency-selective fading with equalization, as shown in [1]. This improvement increases with τ until $\tau = T$ and then remains constant since the two paths are resolvable for $\tau \geq T$. The figure also shows that there is some improvement even if only the interference has frequency-selective fading, but the best improvement occurs when both the desired and interfering signals have frequency-selective fading. A large portion of the maximum possible improvement is obtained when $\tau = 0.7T$. Fig. 8 shows that for $D = 0.382\lambda$, the degradation with correlation increases with frequency-selective fading. As before, however, with $D = 3.82\lambda$, the performance is not degraded until Δ is less than about 20°.

Thus, correlation degrades the performance of an adaptive array that combats fading, suppresses interference, and equalizes frequency-selective fading somewhat more than an array that only combats fading. Correlation up to 0.5 causes little degradation, but higher correlation significantly decreases performance. Although our results show that this degradation increases with the number of antennas, these results are for a linear array, which causes all fading to be highly correlated when signals arrive from endfire, i.e., as $\phi \rightarrow 90°$. Since this problem can be reduced when $M > 2$ by not arranging the antennas linearly, we may be able to avoid this increase in degradation with the number of antennas. However, in all cases, increased antenna spacing reduces the Δ at which degradation occurs.

V. Conclusions

In this paper, we have investigated the effect that fading correlation has on the performance of an adaptive array in a digital mobile radio system. We described a mathematical model of local scattering around the mobile where the received multipath signals arrive at the base station within a given beamwidth and derived a closed-form expression for the correlation as a function of antenna spacing. Monte Carlo simulation results show that the degradation in performance with correlation in an adaptive array that combats fading, suppresses interference, and equalizes frequency-selective fading is only slightly larger than that for combating fading alone, i.e., with maximal ratio combining. This degradation is small even with correlation as high as 0.5. Our results show that, with an antenna spacing of four wavelengths, there is little performance degradation as long as the beamwidth of the received signals is greater than 20°. This tolerable beamwidth can be reduced even further by larger antenna spacing since this beamwidth is inversely proportional to the antenna spacing.

Appendix
Derivation of Closed-Form Expressions for R_{xx} and R_{xy}

The most fundamental description of a linear, quasistationary, multipath medium in wireless systems employing antenna arrays is the impulse response from user "i" to array element output "j." Such a typical impulse response can be represented as the superposition of a large number of impulses,

$$h(t) = \sum_n g_n \delta(t - t_n) \tag{A-1}$$

where the g_n's and the t_n's are the strengths and delays of the possible paths. Clearly, in a time varying situation, these parameters will depend on time. In a system of N users and M antenna elements, we must describe $N \times M$ such responses. Thus, if the input to the medium of a typical user is $s(t)e^{i\omega_o t}$, where ω_o is the angular carrier frequency, the output of a typical antenna element becomes,

$$s_o(t) = e^{i\omega_o t} \sum_n g_n s(t - t_n) e^{-i\omega_o t_n}. \tag{A-2}$$

Following the seminal work of Turin [11], the set of all t_n's is partitioned into L disjoint sets $A_\ell, \ell = 1 \dots L$. With each set A_ℓ, we associate a representative delay τ_ℓ such that $t_n \epsilon A_\ell$ if $s(t - t_n) \approx s(t - \tau_\ell)$. In other words, the differences $\tau_\ell - t_n$ are much smaller than the reciprocal bandwidth of $s(t)$.

With these approximations in mind, we rewrite (A-2) in the form

$$s_o(t) = e^{i\omega_o t} \sum_{\ell=1}^{L} s(t - \tau_\ell) \sum_{n_\ell} g_n e^{-i\omega_o t_n} \tag{A-3}$$

where n_ℓ is the set of integers such that $t_n \epsilon A_\ell$. Denoting $\sum_{n_\ell} g_n e^{-i\omega_o t_n} = b_\ell$ and taking Fourier transforms of both sides of (A-3), we obtain the standard L-ray, or frequency-selective multipath description of fading channels,

$$S_o(\omega) = S(\omega_o + \omega) \sum_{\ell=1}^{L} b_\ell e^{i(\omega_o + \omega)\tau_\ell}. \tag{A-4}$$

Thus, a typical baseband-equivalent, frequency transfer characteristic from user "k" to antenna element "j" can be represented in the form

$$c_{kj}(\omega) = \sum_{\ell=1}^{L} b_\ell^{kj} e^{i\omega\tau_\ell^{kj}}, k = 1, \dots N, j = 1 \dots M. \tag{A-5}$$

For this model to be useful, a statistical characterization of the set of $M \times N$ frequency functions $c_{kj}(\omega)$ must be provided. In our application, we shall assume that the terms in the various sums defining b_ℓ's are random quantities and so it is reasonable to assert that the b_ℓ's are complex random variables. Furthermore, we assume that there are large number of terms in each sum and that each sum includes different random terms and, consequently, from the central-limit theorem, the b_ℓ's, $\ell = 1 \dots L$, may be regarded as i.i.d. complex, zero-mean, Gaussian random variables. If we let $\omega_o t_n = \theta_n$ in the sums defining b_ℓ, we write the real and imaginary parts as

$$b_\ell = \sum_{n_\ell} g_n e^{-i\theta_n} = \sum_{n_\ell} g_n \cos\theta_n + i \sum_{n_\ell} g_n \sin\theta_n = x_\ell + iy_\ell. \tag{A-6}$$

Now, it is reasonable to regard $\theta_n \text{modulo} 2\pi$ as i.i.d. uniformly distributed random variables with the consequence that x_ℓ and y_ℓ are now independent and so $|b_\ell|$ is Rayleigh distributed and $\underline{/b_\ell}$ is uniform. This is then the rationale for regarding $c_{kj}(\omega)$ in (A-5) as a complex Gaussian process in the frequency domain. For our application, the correlation among the elements of c_{kj}'s is of paramount importance.

In order to facilitate the evaluation of these parameters, we must return to the basic definition of the b_ℓ's in (A-6). We begin by considering the following geometrical model. This entails placing the users and the antenna array in a reasonable geometrical relationship. Without loss of generality assume that the antenna array is linear with M elements with identical spacing, D, between elements. We label the elements in ascending order. Users are located at arbitrary angles and distances with respect to the antenna array as depicted in Fig. 2. With each user, we associate a scattering angle of size 2Δ. This implies that all subpaths from the user to the antenna array are restricted to emanate from within this angle.

We now derive the correlations among array elements for a single user by assuming plane waves at the array. This is a reasonable assumption when users and antenna array are separated by many wavelengths.

Suppose the reference wavefront plane coincides with element 1 (see Fig. 2). Then, the wave arriving at element 2 suffers a delay relative to the first element,

$$\tau = \frac{D}{c} \sin\phi, |\phi| \le \pi \tag{A-7}$$

and $(n-1)\tau$ at element "n."

Thus, if we denote the output signals at antenna elements "k" and "j" by $s_{ok}(t)$ and $s_{oj}(t)$, respectively, due to the transmission of a signal of the form, $s(t)e^{i\omega_o t}$, located at an angle ϕ, we can write

$$s_{ok,j}(t) = e^{i\omega_o t} \sum_{\ell=1}^{L} s(t - \tau_\ell) b_\ell^{(k,j)} \tag{A-8}$$

where

$$b_\ell^{(\alpha)} = \sum_{n_\ell} g_n e^{i\theta_n - i\omega_o(\alpha-1)\frac{D}{c}\sin\phi_n}, \alpha = 1, \dots, M$$

where ϕ_n is the angle of arrival of the n-th ray.

As we have already argued, the $b_\ell^{(\alpha)}$'s are complex i.i.d. Gaussian random variables associated with array numbers α, and therefore the sought-after correlations are determined by each $b_\ell^{(\alpha)}$ and different α's. Thus, we seek the correlation coefficients between the following random variables:

$$b_\ell^{(k)} = x_\ell^{(k)} + iy_\ell^{(k)}, k = 1, \dots M$$

and

$$b_\ell^{(j)} = x_\ell^{(j)} + iy_\ell^{(j)}, j = 1, \dots M$$

where

$$x_\ell^{(\alpha)} = \mathrm{Re} b_\ell^{(\alpha)} \tag{A-9}$$

and

$$y_\ell^{(\alpha)} = \mathrm{Im} b_\ell^{(\alpha)}, \alpha = 1, 2, \dots M.$$

We note that since the θ_n's are i.i.d. uniform, the real and imaginary parts of $b_\ell^{(\alpha)}$ are independent for any α. We now calculate for any α

$$E\left[x_\ell^{(\alpha)}\right]^2 = E\left[y_\ell^{(\alpha)}\right]^2 = \frac{1}{2} \sum_{n_\ell} E\left[g_n^2\right]. \tag{A-10}$$

It is now straightforward to calculate the four correlation coefficients

$$E\left[x_\ell^{(k)} x_\ell^{(j)}\right] = E\left[y_\ell^{(k)} y_\ell^{(j)}\right] = \frac{1}{2} \sum_{n_\ell} E\left[g_n^2 \cos\left[(k-j) 2\pi \frac{D}{\lambda} \sin\phi_n\right]\right] \tag{A-11}$$

and

$$E\left[x_\ell^{(k)} y_\ell^{(j)}\right] = -E\left[x_\ell^{(j)} y_\ell^{(k)}\right] = \frac{1}{2}\sum_{n_\ell} E\left[g_n^2 \sin\left[(k-j)2\pi\frac{D}{\lambda}\sin\phi_n\right]\right] \tag{A-12}$$

where

$$\omega_o\frac{D}{c} = 2\pi f_o\frac{D}{c} = 2\pi\frac{D}{\lambda}.$$

According to our hypothesis, there are a large number of terms in the sums indicated in (A-10) and (A-11) and if we make the additional physically reasonable assumption that the ϕ_n's are dense in the range $(\phi-\Delta, \phi+\Delta)$, the sums can be expressed as integrals of the form, independent of ℓ,

$$R_{xx}(k-j) = R_{yy}(k-j) = E\left[x_\ell^{(k)} x_\ell^{(j)}\right] = E\left[y_\ell^{(k)} y_\ell^{(j)}\right] \tag{A-13}$$

$$= \frac{1}{2\Delta}\mathrm{Re}\int_{\phi-\Delta}^{\phi+\Delta} \sigma^2(\beta) e^{i\frac{2\pi D}{\lambda}(k-j)\sin\beta} d\beta$$

and

$$R_{xy}(k-j) = -R_{yx}(k-j) = E\left[x_\ell^{(k)} y_\ell^{(j)}\right] = -E\left[x_\ell^{(j)} y_\ell^{(k)}\right] = \frac{1}{2\Delta}\mathrm{Im}\int_{\phi-\Delta}^{\phi+\Delta} \sigma^2(\beta) e^{i\frac{2\pi D}{\lambda}(k-j)\sin\beta} d\beta \tag{A-14}$$

where the density function of the returned strengths $\sigma^2(\phi)$ must satisfy

$$\frac{1}{2\Delta}\int_{\phi-\Delta}^{\phi+\Delta} \sigma^2(\beta) d\beta \to \frac{1}{2}\sum_{n_\ell} E\left[g_n^2\right]. \tag{A-15}$$

Making the reasonable assumption that this density function is a constant over the angle segments, we then obtain the relationship

$$\sigma^2 = \frac{1}{2}\sum_{n_\ell} E\left[g_n^2\right] = \frac{1}{2}E\left[|b_\ell|^2\right], \text{for all } \ell \tag{A-16}$$

which is consistent with the definitions in (A-8). Now, by making use of the well-known series representations,

$$\cos(z\cos\theta) = J_o(z) + 2\sum_{m=1}^{\infty} J_{2m}(z)\cos(2m\theta)$$
$$\sin(z\sin\theta) = 2\sum_{m=0}^{\infty} J_{2m+1}(z)\sin[(2m+1)\theta] \tag{A-17}$$

where the J_m's are Bessel Functions of integer order and

$$z = 2\pi\frac{D}{\lambda}, \tag{A-18}$$

we can integrate (A-13) and (A-14) and obtain the following convenient formulas for the desired correlation coefficients:

$$\tilde{R}_{yy}(k-j) = \tilde{R}_{xx}(k-j) = J_o(z(k-j)) + 2\sum_{m=1}^{\infty} J_{2m}(z(k-j))\cos(2m\phi)\frac{\sin(2m\Delta)}{2m\Delta} \tag{A-19}$$

and

$$\tilde{R}_{xy}(k-j) = -\tilde{R}_{yx}(k-j) = 2\sum_{m=0}^{\infty} J_{2m+1}(z(k-j))\sin[(2m+1)\phi]\frac{\sin((2m+1)\Delta)}{(2m+1)\Delta} \tag{A-20}$$

where the normalized R's are defined as $\tilde{R} = R/\sigma^2$. It can be readily checked that

$$\tilde{R}_{xy}(0) = \tilde{R}_{yx}(0)$$

and

$$\tilde{R}_{xx}(0) = \tilde{R}_{yy}(0) = 1,$$

as they must be for "physically consistent" considerations.

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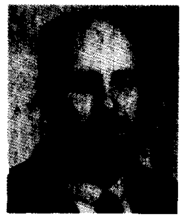

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