Simulation Study of the Capacity Bounds in Cellular Systems

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Abstract

In this paper, we investigate the capacity of cellular systems. In particular, we study how the reuse factor can be improved given the knowledge of the mobiles' locations; i.e., we evaluate the minimum number of channels required to support a cellular infrastructure with a given number of mobiles in each cell. We assume that the mobiles' locations are sampled from the uniform random distribution or are fixed on a uniform grid. Moreover, we show the effect of a number of parameters, such as the number of mobiles per cell, the minimum allowable signal-to-interference ratio, and limited knowledge of mobile location. The assumption of a single interferer, used in our study, is also justified.

1 Introduction

A large body of research has been published on the performance of cellular systems (e.g., [1] - [6]). Most of these papers present and analyze schemes in which no knowledge about the mobiles' locations, besides the mobiles' associated base-stations, is taken into the account. Thus, a channel is allocated to a cell, rather than a mobile. In this work, we assume that there is a mechanism by which mobiles can measure the amount of interference that they receive on different channels. Thus, in our work, the allocation of channels is to mobiles rather than cells. We show the amount of improvement that such a scheme carries.

In fixed channel allocation (FCA), a channel assigned to a cell can be reused according to some (fixed) cell reuse pattern ([1]). For example for cell reuse pattern of 7, a channel used in the cell A is not reused at any other cell whose distance to the cell's A base-station is closer than about three cell "radii." Similarly, for cell reuse pattern of 3, the corresponding distance is 2 "radii."

Because, at any time, some cells may require more capacity than others, the FCA suffers from inefficient use of the radio spectrum. To correct this limitation of FCA, Dynamic Channel Allocation (DCA) schemes have been proposed.[1] In fact, a multiplicity of DCA schemes have been investigated by various researchers, showing the advantage of one scheme over another under some traffic and other assumptions (e.g., [5, 7, 8, 9, 10]). The traffic adaptation based DCA schemes rely on the idea that, instead of dedicating the channels to cells, the channels are "placed" in a pool, and are allocated on demand to the cells based on some allocation rules.

In both FCA and the traffic adaptation DCA, the channels are usually allocated to cells based on the assumption that a mobile may be located anywhere within the boundary of that cell. Thus, for both the schemes, the "packing"[2] of channels is not maximal. Theses schemes suffer from the fact that the fixed reusability factor may be too pessimistic; e.g., mobiles may not interfere with each other even if they're in adjacent cells and the reuse pattern corresponds to distance of 2 radii. In the interference adaptation DCA schemes mobiles measure the amount of interference to determine the usability of a channel. Such algorithms can achieve maximal packing. An example of a system based on this principle is the DECT standard ([11]).

The maximum packing corresponds to the minimum number of channels required to support a given amount of traffic in each cell. In other words, a maximal packing for the specific mobile distribution means that whatever the allocation, the computed number of channels is the minimum number of channels that must be allocated to provide connection to all the mobiles – the number of channels with maximum packing is a lower bound on the number of channels required in any cellular system.

The minimum number of channels under the maximum packing condition is a static measure. In other words, when the mobiles move, the particular assignment may not be valid anymore and may need to be recomputed. This may lead to reassignment of the channels of many (if not all) of the mobiles in the system. We are investigating the effect of mobility on the required reassignment frequency.

In general, the bound will vary with the location of the mobiles. However, our results show that with randomly-located mobiles (the random locations drawn from the uni-

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[1] We distinguish here between DCA schemes that respond to traffic changes (traffic adaptation) and to changes in the level of measured interference (interference adaptation). For example, see [6].

[2] The channel "packing" refers to the areas where a channel cannot be reused and how closely these areas are spaced.

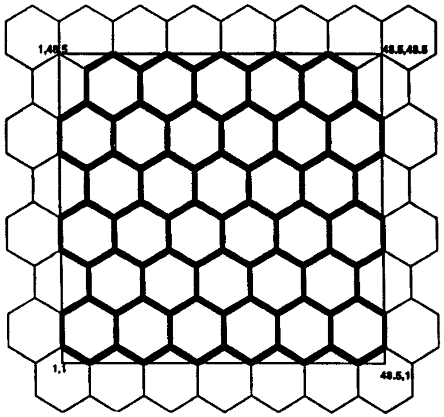

Figure 1: Cells structure

form distribution), the variation in the bound for different realizations of the random locations is relatively small. Therefore, we use Monte Carlo simulation to determine the bound for a few realizations and use these results to obtain an approximate bound for most cases of randomly located users (i.e., small probability of blocking).

In practice, we may not need to assign channels to all mobiles, but can block some small percentage of arriving calls. Here we ignore this fact, which would result in somewhat fewer channels being required.

Also, because of the time-variations in the traffic, the required number of channels will also vary. Using the results presented here and knowing the traffic variations (or distribution), one may readily determine the distribution of the required number of channels with the traffic variations.

This paper is organized as follows: In the next section, we describe the model of the cellular network and list the major assumptions used throughout this work. In section 3, we show the effect of the different parameters on the results of the maximum packing strategy. For example, we consider the effect of channel allocation based on the distance of the mobile from the base station, without knowledge of the actual location of the mobile. In subsection 3.6, we provide justification for the single interfer assumption. Finally, in section 4, we summarize and conclude the study.

2 Assumptions, Parameters, and Methodology

The following model is used throughout work (unless specifically indicated otherwise):

1. The cellular structure is composed of 33 internal cells, as shown in Figure 1, surrounded by additional cells. The purpose of the additional cells is to eliminate the "boundary effects," which would lead to too optimistic results (too small number of required channels). In other words, the number of channels required for this structure should be approximately the same as the number required for an arbitrary large cellular structure. A base-station is located in the middle of each cell, and serves the mobiles that are located within the boundaries of the cell.

2. Except section 3.4, the mobiles are randomly distributed within each cell. However, in each case, the number of mobiles per cell is a constant (i.e., not a random) number.[3]

3. The *single interferer* assumption: In our study we assume that there is only one interfering link (i.e., with the same assigned channel) to a communication link under study. In reality, there may be several other links communicating on the same channel and contributing interference to the link in question. However, as shown in section 3.6, when a single interferer is used to calculate channel allocation, there is only a small probability that the resulting total interference of all the links exceeds the designed-for interference level. Thus, the *single interferer* assumption is justified.

4. The up link and the down link channels are paired; e.g., if a specific up link channel is available for communication between a mobile and its base-station, but its paired down link channel suffers from too much interference, this pair cannot be used.

5. We do not consider *shadow fading*. Our model accounts for Rayleigh fading and propagation loss only.

6. Both the base-stations and the mobiles are equipped with omni-directional antennas.

7. Except for section 3.5, there is no power control at the mobiles or at the base stations.

The parameters used in the study are:

- n - number of mobiles in each cell
- c - the total number of cells
- N - the total number of mobiles ($= n \cdot c$)
- α - the *interference radius* - the required ratio between the distance from the base-station to the interferer and the distance from the base-station to the mobile
- P_{BS} - the power transmitted by a base-station (we assume all the base-stations transmit the same power level)
- P_M - the power transmitted by a mobile (we assume all the mobiles transmit the same power level)
- S - the signal's received power
- I - the level of received interfering signal
- r - the power of radio signal loss (r=2 for free space propagation without the multipath effect; for mobile radio r=3.8 is usually assumed)
- N_c - number of channels required to accommodate the n users under given conditions
- x - the distance of a mobile to its base-station
- d - the distance of a mobile to another base-station (other than its base-station)

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[3] The reason for keeping the number of mobiles per cell constant is to produce results that can be compared with other studies.

The study evaluates the minimum number of channels (referred to here also as colors) assigned to the mobiles under given operating conditions, such that given interference conditions are satisfied. The operating conditions refer to the knowledge or lack of knowledge of the location of the mobiles. The interference conditions refer to the acceptable level of interference, so that two mobiles can be assigned the same channel.

In this work, we consider FDMA/TDMA assignment. Two different TDMA slots are considered separate channels and we assume here that such two channels do not suffer from cross interference.

The number of channels is evaluated by constructing a matrix (referred to here as *compatibility matrix* of dimension $N \times N$); i.e., each mobile in the whole system is evaluated with each other mobile to determine whether the two mobiles can be assigned the same channel (i.e., whether they are "compatible") based on the interference conditions. A graph is then composed, where each mobile in the system corresponds to a vertex in the graph. Two vertices are interconnected by an edge in the graph, if and only if the two mobiles represented by the two vertices are "incompatible"; i.e., they cannot be assigned the same channel. A set of graph coloring algorithms is then employed to find the minimum number of colors to color the vertices in the graph, such that no two vertices interconnected by an edge are colored in the same color. Each color corresponds then to a channel assigned to the mobiles that the vertices colored in that color represent. Thus, the number of colors required equals the number of channels. (This problem can also be posed as finding the minimum number of cliques[4] that cover all the vertices in the complementary graph.[5] A clique in the complementary graph, then, corresponds to a channel.) For a small number of vertices, finding the number of colors may not be a considerable obstacle. However, to obtain any meaningful statistics, instances with large number of mobiles need to be evaluated. Thus, since the graph coloring problem is an NP-complete problem ([12]), finding the number of colors can be a complex task as is the problem of finding the minimum number of cliques. In this work, we thus use an algorithm suggested by [13] that, while not finding the exact minimum number of colors required, is remarkably effective at finding good upper bounds on that number for the types of graphs we create. We then complement our upper bounds with lower bounds determined by heuristics for the maximum clique problem, thus pinning the precise value within a narrow range, and indeed often determining it exactly.

Other approaches to cope with the large complexity of the problem have been proposed. In particular, simulated annealing and neural networks were used ([14]).Our algorithm is significantly faster than the simulated annealing and neural net approaches and finds bounds that are typically just as good, or, in the latter case significantly better.

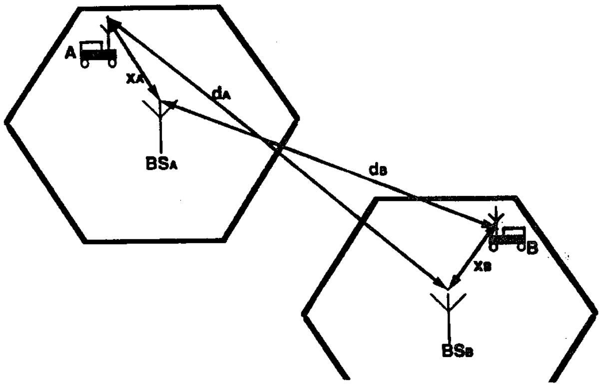

Figure 2: Interference distances

Such schemes usually suffer from the uncertainty whether the obtained solution is, in fact, optimal and what is the error from the optimal solutions. Our "brute force" scheme presented here provides either an 'exact' solution or tight bounds within which the optimal result exists.

In [6], an attempt to compare the FCA with interference adaptation DCA scheme through analytical methods has been made. However, the results for interference adaptation DCA scheme are shown as two quite loose bounds. As mentioned before, our study provides much more specific results.

2.1 The Interference Conditions

In general, the signal-to-interference (SIR) ratio determines the ability of the receiver to recover the signal of power S in the presence of interference of power I. We assume that there is some minimum SIR, termed SIR_{min}, required for sufficient system operation. Thus, in a system in which the desired signal of power P is transmitted from a distance x and the interference signal of the same power is transmitted from distance d, the SIR is:

$$\mathrm{SIR} = \frac{S}{I} = \frac{\frac{P}{x^r}}{\frac{P}{d^r}} = (\frac{d}{x})^r \geq \mathrm{SIR}_{min}. \qquad (1)$$

We define α to be:

$$\alpha = \sqrt[r]{\mathrm{SIR}_{min}}. \qquad (2)$$

Thus to achieve sufficient performance (i.e., $\mathrm{SIR} \geq \mathrm{SIR}_{min}$):

$$\frac{d}{x} \geq \alpha. \qquad (3)$$

Assume there are two mobiles, one in each cell, as shown in Figure 2. If the two mobiles are assigned the same up link channel and the two base-stations are assigned the

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[4] A *clique* is a subgraph in which every two vertices are interconnected by an edge.

[5] The complementary graph is a graph that has edge joining two nodes if and only if they were not so joined in the original graph.

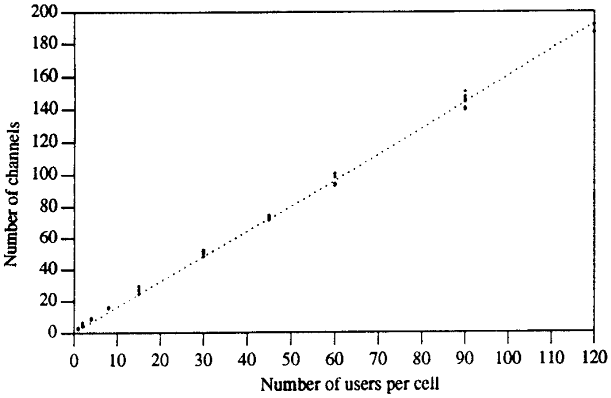

Figure 3: Number of channels as a function of mobile density

same down link channel, then the SIR of the base-station$_A$ to mobile A communication, resulting from interferering communication between base-station$_B$ to mobile B is:

$$\mathrm{SIR}_A = \frac{\frac{P_{BS}}{(x_A)^r}}{\frac{P_{BS}}{(d_A)^r}} = (\frac{d_A}{x_A})^r \tag{4}$$

and the SIR of the mobile A to base-station$_A$ communication, resulting from interfering communication between the mobile B and the base-station$_B$ is:

$$\mathrm{SIR}_{BS_A} = \frac{\frac{P_M}{(x_A)^r}}{\frac{P_M}{(d_B)^r}} = (\frac{d_B}{x_A})^r \tag{5}$$

Similarly,

$$\mathrm{SIR}_B = \frac{\frac{P_{BS}}{(x_B)^r}}{\frac{P_{BS}}{(d_B)^r}} = (\frac{d_B}{x_B})^r \tag{6}$$

$$\mathrm{SIR}_{BS_B} = \frac{\frac{P_M}{(x_B)^r}}{\frac{P_M}{(d_A)^r}} = (\frac{d_A}{x_B})^r \tag{7}$$

Since each one of the SIR's must satisfy the condition that SIR$\geq$ SIR$_{min}$, equations (4) – (7) correspond to the following set of conditions:

$$d_A \geq \alpha \cdot x_A d_B \geq \alpha \cdot x_A d_B \geq \alpha \cdot x_B d_A \geq \alpha \cdot x_B \tag{8}$$

There are n mobiles per cell, randomly located throughout the cell. The random location is drawn from the uniform distribution. The conditions (8) are checked for every pair of mobiles and two mobiles are declared compatible if the conditions (8) are satisfied. Otherwise, the mobiles are incompatible. The results are arranged in the compatibility matrix, which is used as an input to our graph-coloring and clique finding algorithms. From these we obtain upper and lower bounds (typically quite tight) on the minimum number of channels required to support communication to/from all the mobiles with SIR greater than SIR$_{min}$.

While knowing the location of each mobile may not be possible, all that is necessary for maximal packing is the knowledge of the signal levels of each mobile's signal at all the base stations and the knowledge of each base station signal at all the mobiles. With such knowledge, the signal-to-interference for an allocation can be determined.

3 Bounds on the Number of Channels

In this section, we show the effect of the various parameters in our system on the minimum number of channels, N_c.

3.1 The Effect of the Number of Mobiles per Cell

The number of required channels, N_c, as a function of mobile density is shown in Figure 3. Four runs were performed for the following number of mobiles per cell: $n = 1$, 2, 4, 8, 15, 30, 45, 60, 90, 120. As demonstrated in this figure, the number of channels increases linearly with the mobile density, measured here in mobiles-per-cell. The results shown in this figure were obtained with $\alpha = 2.0$, which corresponds to reuse pattern of 3. The slope of the curve in Figure 3 is about 1.6. Thus, we have shown that for $\alpha = 2$, the reduction in the number of channels offered by maximal packing vs. fixed reuse pattern is approximately a factor of 2; i.e., the maximal packing requires only about half as many channels as FCA. Moreover, because of the linear behavior, many of our results that follow are independent of the actual number of mobiles per cell. Note that, because the number of mobiles per cell is constant in our model, traffic adaptation DCA has no advantage over FCA.

3.2 The Effect of the Interference Radius

The two curves in Figure 4 show the dependence of the number of required channels on the interference radius, α, for the maximal packing scheme. The results were obtained for 90 mobiles per cell. This figure also shows the relative reduction of the number of channels with the maximal packing scheme, compared with the following maximally packed FCA scheme. The maximally packed FCA scheme assumes that the channel assignment is fixed to the cells, but is not necessarily repetitious. This is in contrast with the "traditional" FCA schemes, where the assignments to the cells are replicated and appear as a repetitious pattern. The numbers for the maximally packed FCA scheme were obtained by running our maximum packing algorithm with a single mobile per cell located on the "circumference" of the cell, at the point that is closest to the circumference of the cell of the interfering mobile (i.e., the distance is the shortest distance between two points, one

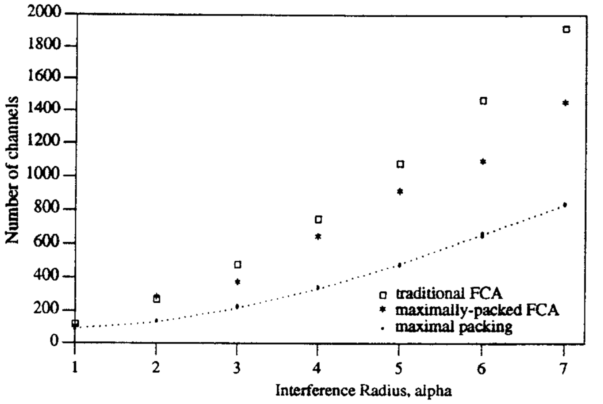

Figure 4: Number of channels as a function of the interference radius

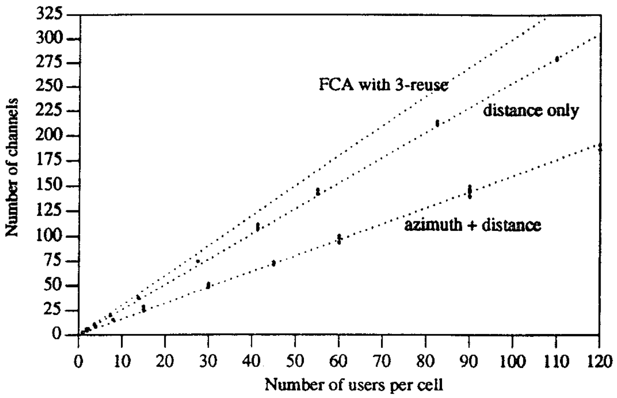

Figure 5: Number of channels as a function of mobile density, when the azimuth is not known

point of the circumference on each cell). For a given interference radius, the minimum number of colors equals the re-use factor.

For comparison purposes, the traditional FCA scheme is also shown in Figure 4. The number of channels for the traditional scheme was obtained by the following formula (taken from [5] with adjustments to reflect our definitions):

$$N_c(\mathrm{FCA}) = \frac{n}{3}(\alpha+1)^2. \qquad (9)$$

As clearly shown in the figure, the actual gain achieved from the maximal packing algorithm increases with the interference radius as compared with the traditional FCA and, compared to the maximally packed FCA, achieves an approximately constant gain of 40% over the range of $\alpha = 2$ to $\alpha = 7$. For example, for interference radii of 2 and 3.6, the corresponding fixed reuse patterns for the traditional FCA are 3 and 7, respectively. The number of channels with the maximal packing is approximately 145 and 230, respectively, and the reduction in the number of channels is, thus, 52% and 67%, respectively. For $\alpha = 2$ and $\alpha = 4$, the maximally packed FCA requires 270 and 630 channels, respectively, while the maximum packing scheme requires 145 and 350 channels, respectively, reducing the number of channels by 46% and 44%, respectively.

3.3 The Effect of No Knowledge of Direction (Azimuth)

By measuring the received power,[6] the base-station may be able to determine the distance of the mobile, without knowing its actual position; i.e., ignorance of the azimuth. When the maximum packing is performed without the knowledge of the azimuth, the gain of the maximum packing is reduced, as shown in Figure 5. Curves "azimuth+distance" and "distance only" represent the cases when the actual location of the mobile is known, and when only the distance of the mobile from the base-station is known, respectively. For comparison purposes, the curve "FCA with 3-reuse" shows the results for FCA with 3-reuse pattern. Results in Figure 5 were obtained with $\alpha = 2.0$. Thus, for example, for 90 mobiles per cell and for $\alpha = 2.0$, the number of required channels with no knowledge of direction is 229, or a reduction of 24%. This is about half of the reduction of 52%, when the direction is known.

When the power levels, rather than the actual mobiles' locations are used (and assuming no shadow fading), no knowledge of the direction is equivalent to the situation where the information about the received power levels is not shared among the base-stations (and among mobiles); i.e., each base-station knows the signal levels of its mobiles only. Similarly, every mobile knows the signal levels from all the base stations, but not other mobiles' levels.

3.4 The Effect of Mobile Distribution within the Cell

The previous results were obtained by assuming random (uniform) distribution of mobiles within the cell. When the mobiles' locations are fixed on a uniform grid, the gain of the maximum packing algorithm is considerably improved, since regions of high mobile concentration ("hot spots") can occur under the random distribution. Obviously, such regions are not present in the fixed uniform distribution case, and the results are shown in Figure 6. As shown in this figure, the slope of the curve is about 1.4, indicating about 15% fewer channels than in the randomly distributed case. The results were obtained with $\alpha = 2.0$. The actual position of the grid with respect to the cellular structure has no noticeable effect with large number of mobiles per cell.

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[6] When power control is performed, additional information on the transmitted power needs to be conveyed to the receiver site.

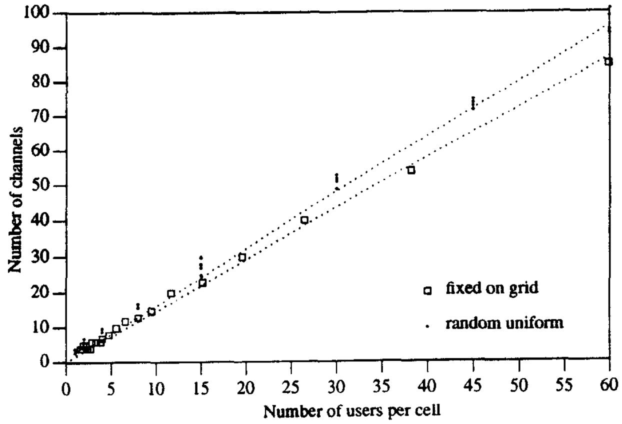

Figure 6: Number of channels as a function of mobile density for uniformly distributed mobiles

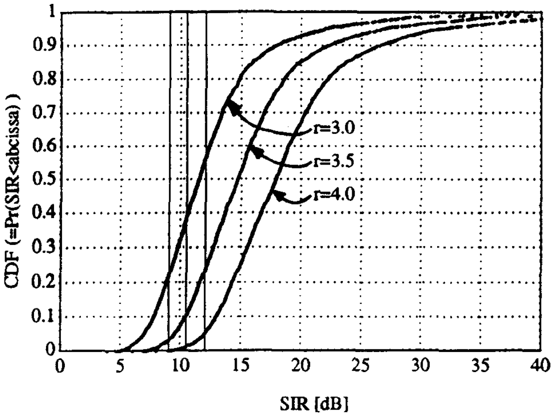

Figure 7: Cumulative probability distribution function of the mobiles' SIR

3.5 The Effect of the Power Control

In this section we show that the power control has no effect on the number of channels, as investigated in this study.

In the power controlled environment, as is most commonly used, mobiles transmit their signal with dynamically adjusted power level, so that a constant power level is received at the base stations; i.e., the effect of loss and fading is compensated. Similarly, the power of base stations' transmission is adjusted so that the signal received at the mobiles is of constant level.

If the power received by a mobile is P_{BS}, the actual power transmitted by a base-station located at a distance x is $P \cdot x^r$. Similarly, if the power received by a base-station from a mobile located at a distance x is P_M, the mobile needs to transmit power of $P \cdot x^r$. Consequently, the equations (4 – 7) can be rewritten as:

$$\mathrm{SIR}_A = \frac{\frac{P_{BS}\cdot(x_A)^r}{(x_A)^r}}{\frac{P_{BS}\cdot(x_B)^r}{(d_A)^r}} = (\frac{d_A}{x_B})^r \tag{10}$$

$$\mathrm{SIR}_{BS_A} = \frac{\frac{P_M\cdot(x_A)^r}{(x_A)^r}}{\frac{P_M\cdot(x_B)^r}{(d_B)^r}} = (\frac{d_B}{x_B})^r \tag{11}$$

$$\mathrm{SIR}_B = \frac{\frac{P_{BS}\cdot(x_B)^r}{(x_B)^r}}{\frac{P_{BS}\cdot(x_A)^r}{(d_B)^r}} = (\frac{d_B}{x_A})^r \tag{12}$$

$$\mathrm{SIR}_{BS_B} = \frac{\frac{P_M\cdot(x_B)^r}{(x_B)^r}}{\frac{P_M\cdot(x_A)^r}{(d_A)^r}} = (\frac{d_A}{x_A})^r \tag{13}$$

Thus the four conditions that the locations of the two mobiles need to fulfill to be assigned the same channel (i.e., to be compatible) are:

$$d_A \geq \alpha \cdot x_B d_B \geq \alpha \cdot x_B d_B \geq \alpha \cdot x_A d_A \geq \alpha \cdot x_A, \tag{14}$$

which are exactly the conditions (8), in different order. Thus we conclude that two mobiles' locations are compatible, regardless of whether power control is performed; i.e., our results are applicable to both the power-controlled and the non-controlled cases. A similar conclusion can be drawn from [15].

3.6 The Effect of the "Single-Interferer" Assumption

In this study, we have assumed a single interferer. This assumption implies that in each case, there is one interferer that is significantly closer than all the other interferers. In this section, we justify the single interferer assumption.

In Figure 7, the probability distribution function of the mobiles' SIR is shown for the cases of r =3.0, 3.5, and 4.0. The probability distribution was obtained for $\alpha = 2.0$ by analyzing the actual assignments generated by the graph-coloring algorithms. In other words, using the assignment obtained from the graph-coloring algorithm, the level of the interference signal at each mobile from all the other mobiles (with the same channel assigned) was calculated and compared with the received signal strength. The statistics of all the mobiles were obtained, resulting in the C. D. F. in Figure 7. For $\alpha = 2.0$, the corresponding SIR_{min}-s for the above three values of r are: 9 dB, 10.5 dB, and 12 dB, respectively. Vertical lines, corresponding to these values are shown in the Figure 7. Considering all the interferers, Figure 7 shows that, when the single interferer assumption is made, the percentages of mobiles with SIR worse than SIR_{min} are: 20%, 10%, and 5%, for r = 3.0, 3.5, 4.0, respectively. We conclude that the results generated using the single interferer assumption contain channel assignments with relatively small percentage of mobiles violating the SIR that the system was originally designed for. Consequently, the error in the minimum number of channels due to the single interferer assumption for $r \geq 3.5$ may be considered negligible.

4 Concluding Remarks

In this paper, we have investigated the performance of the maximum packing algorithm for cellular structure. Our results are innovative in the sense that the channel assignments are done based on the knowledge (or limited knowledge) of the mobiles' locations. Thus, the cellular structure does not, by itself, limit the reuse of channels. An example of such a system is one in which the mobiles (and the base stations) measure the amount of interference to determine the available channels. The assignments are then done based on these measurement results.

Our investigation used heuristics for graph-coloring and clique-finding to obtain tight bounds on the minimum number of channels needed in each situation. The heuristics were chosen both for their speed and for the quality of bounds they yielded, and in these respects seemed to significantly outperform such other methods as simulated annealing and neural nets.

The results of this study indicate that the maximal packing scheme presented here can reduce the number of channels by nearly a factor of 2, for interference radius of 2.0. More specifically, approximately 40% fewer channels are required in the maximum packing scheme than in maximally-packed (i.e., not necessarily repetitious) FCA, independent of the interference radius. Furthermore, the improvement with respect to the (traditional) FCA scheme is even larger and increases with the interference radius.

When only the distance (or power level) between each mobile and its nearest base station is known, the reduction in the number of channels is somewhat modest, about 25% for interference radius of 2.0. More uniform mobile distribution reduces the number of required channels, since mobile clustering tends to reduce the channel reuse. We found about a 15% difference in the number of channels between the random (uniform) and fixed (on square grid) cases (for interference radius of 2.0).

We have demonstrated that full power control that compensates for power loss has no effect on the number of channels and that the assumption of a single interferer, as used in our study, was justified, showing that only about 10% of mobiles experience worse than designed for signal-to-interference ratio as the result of the single interferer assumption (assuming r=3.5).

The schemes shown here may be of particular interest to PCN, where the user density may be considerable and efficient spectrum reuse may be crucial.

5 Acknowledgement

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