The Diversity Gain of Transmit Diversity in Wireless Systems with Rayleigh Fading

Jack H. Winters, *Fellow, IEEE*

***Abstract*— In this paper, we study the ability of transmit diversity to provide diversity benefit to a receiver in a Rayleigh fading environment. With transmit diversity, multiple antennas transmit delayed versions of a signal to create frequency-selective fading at a single antenna at the receiver, which uses equalization to obtain diversity gain against fading. We use Monte Carlo simulation to study transmit diversity for the case of independent Rayleigh fading from each transmit antenna to the receive antenna and maximum likelihood sequence estimation for equalization at the receiver. Our results show that transmit diversity with M transmit antennas provides a diversity gain within 0.1 dB of that with M receive antennas for any number of antennas. Thus, we can obtain the same diversity benefit at the remotes and base stations using multiple base-station antennas only.**

***Index Terms*—Diversity gain, Rayleigh fading, transmit diversity, wireless communications.**

I. Introduction

THE EFFECT of multipath fading in wireless systems can be reduced by using antenna diversity. In many systems, though, additional antennas may be expensive or impractical at the remote or even at the base station. In these cases, transmit diversity can be used to provide diversity benefit at a receiver with multiple transmit antennas only. With transmit diversity, multiple antennas transmit delayed versions of a signal, creating frequency-selective fading, which is equalized at the receiver to provide diversity gain.

Previous papers have studied the performance of transmit diversity with narrowband signals [1]–[5] using linear equalization, decision feedback equalization, maximum likelihood sequence estimation (MLSE), and spread-spectrum signals [6]–[8] using a RAKE receiver. Monte Carlo simulation results [3], [5] showed that, using MLSE with narrowband signals, the diversity gain with two transmit antennas was similar to that with two receive antennas using maximal ratio combining.[1] However, with three transmit antennas, the diversity gain was less than that of three-antenna receive diversity at high bit-error rates (BER's).

In this paper, we study the diversity gain of transmit diversity with ideal MLSE and an arbitrary number of antennas and compare the results to receive diversity with maximal ratio combining. We consider binary-phase-shift-keyed (BPSK) modulation with coherent detection and assume independent Rayleigh fading between each transmit antenna and the receive antenna with the delay between the transmitted signals such that the received signals are uncorrelated. This comparison of M-antenna transmit diversity to receive diversity is shown to be the same as comparing ideal MLSE to the matched filter bound with an M-symbol-spaced impulse response. With a double impulse response, MLSE can achieve the matched filter bound for all channels [9]. However, with more than a double impulse response, there exist channels for which MLSE cannot achieve the matched filter bound [9]. Using Monte Carlo simulation with Rayleigh fading, we determine the probability distribution of the Euclidean distance between MLSE and the matched filter bound and the resulting degradation in performance. Although this degradation can be several dB for some channel instances,[2] our results show that large degradation occurs with low probability and, when it does occur, is usually on channels with good performance. Therefore, the degradation has little effect on the distribution of the BER with Rayleigh fading. Specifically, our results for 2–30 antennas show that transmit diversity can achieve diversity gains within 0.1 dB of receive diversity. Thus, we can obtain the same diversity benefit at the remotes and base stations using multiple base-station antennas only.

In Section II, we describe transmit diversity and cast the evaluation of performance into a comparison of MLSE to the matched filter bound. We present results for the distribution of the Euclidean distance between MLSE and the matched filter bound in Section III and discuss other issues concerning transmit diversity in Section IV. A summary and conclusions are presented in Section V.

II. Description of Transmit Diversity

Fig. 1 shows a block diagram of transmit diversity with M transmit antennas in a wireless system. The digital signal $s(t)$ is transmitted by each antenna with a D-s delay between each antenna. The total transmit power P_T is equally divided among all antennas, i.e., the transmit power for each antenna is given by

$$P_{T_i} = P_T/M, \qquad i = 1, \ldots, M. \tag{1}$$

Manuscript received May 20, 1994; revised August 3, 1995.

The author is with AT&T Labs—Research, Newman Springs Lab, Red Bank, NJ 07701-7033 USA.

Publisher Item Identifier S 0018-9545(98)00105-4.

[1] Note that with transmit diversity, we obtain a diversity gain against fading because of the different fading channels between each transmit and receive antenna, but do not get the antenna gain of receive diversity, i.e., an M-fold increase in receive signal-to-noise ratio with M antennas. With multipath fading, this diversity gain is substantially more than the antenna gain.

[2] By channel instance, we mean a sample from the ensemble of channels with independent Rayleigh fading. Although the type of channel we are considering is the Rayleigh fading channel, in the remainder of this paper we will refer to a specific channel instance as a channel.

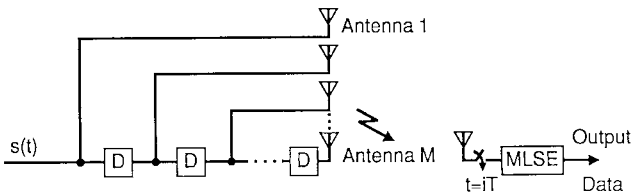

Fig. 1. Block diagram of transmit diversity.

We will assume independent flat Rayleigh fading between each transmit and the receive antenna. Note that the assumption of independent fading between antennas is the same as that required for full diversity gain with receive diversity.

The delay between antennas is chosen so that the signals transmitted by each antenna are uncorrelated, i.e.,

$$E[s(t)s(t+D)] = 0. \tag{2}$$

For our analysis, we will assume that the transmitted symbols are independent and that the transmit and receive filters do not cause intersymbol interference in the received signal. With these assumptions, including the flat fading assumption, a delay of at least one symbol period T $(D \geq T)$ is required for uncorrelated receive signals from each transmit antenna. We therefore will consider the case of $D = T$, since a shorter delay results in correlation between the transmitted signals, which reduces the diversity gain of transmit diversity, while a longer delay increases the complexity of the equalizer at the receiver without improving the diversity gain. When delay spread is present in the channel, i.e., without flat fading, a longer delay is needed for uncorrelated received signals. For example, with a delay spread of $\pm T$, $D \geq 2T$ is needed to achieve uncorrelated signals and the maximum diversity gain at the receiver. Some results of effect of delay on the diversity gain of transmit diversity with delay spread are presented in [2].

At the receiver, white Gaussian noise is added to the received signal, the received signal is sampled at the symbol rate, and the transmitted symbols are determined by MLSE. Here, we consider ideal MLSE, i.e., infinite length MLSE with perfect channel knowledge.

To simplify the problem, let us consider BPSK modulation with coherent detection. Thus, the transmitted signal can be considered as a real binary signal. Our analysis below can be extended to the case of complex multilevel signals (i.e., quadrature amplitude modulation) as well.

With the above assumptions, the transmit diversity system of Fig. 1 can be modeled as the discrete time system (as in [9]) shown in Fig. 2, where the input is an independent binary random sequence $x = \{x_i\}$ with outcomes ± 1 equally likely, and the transmitter and channel impulse response (the system response) is given by

$$h = \cdots 00 h_0 h_1 \cdots h_{M-1} 00 \cdots. \tag{3}$$

With independent Rayleigh fading, the h_i's, $i = 0, \ldots, M-1$, are independent complex, Gaussian random variables with zero mean and variance P_T/M. The noise $n = \{n_i\}$ is a sequence of independent complex Gaussian random variables with zero mean and variance N_0. The sequence input to MLSE $y = \{y_i\}$ is then $y = x * h + n$, where $*$ denotes convolution.

With MLSE, the BER for a given channel is approximately given by the probability of minimum distance error events [10]. This approximation is accurate at least for low BER's. Specifically, the BER is given by [10]

$$\text{BER} \approx 1/2\,\text{erfc}\left(\sqrt{\frac{d_{\min}^2}{N_0}}\right) \tag{4}$$

where the minimum distance over all possible error events is given by

$$d_{\min}^2 = \min_{\varepsilon} ||h * \varepsilon||^2. \tag{5}$$

In (5), $|| \, ||^2$ denotes the l_2 norm (sum of the squares of the elements) and

$$\varepsilon = \cdots 00 \varepsilon_0 \cdots \varepsilon_l 00 \cdots, \; \varepsilon_k = \pm 1 \tag{6}$$

denotes an error event of length l, where ε_k is the kth error symbol.

Since there are an infinite number of possible error sequences, to determine the sequence with $d_{\min}$, we must use a search technique that limits the number of error sequences to be examined. One such technique, using tree pruning, is described in [11]. That paper considers real binary (as well as multilevel) signals with real channels rather than complex channels. Therefore, we modified the program used in [11] for complex channels. In addition, we eliminated the "half test," which assumed a symmetrical impulse response, which we do not have, in general. Eliminating this test greatly increased computation time, but even with $M = 30$, the program took less than 1 min on a SPARC10 to find the minimum distance for a given channel.

Now, the matched filter bound for this system is the squared distance of an isolated single bit-error event. Thus, from (5), this distance is given by

$$d_{\min|_{\text{MFB}}}^2 = \sum_{i=0}^{M-1} |h_i|^2. \tag{7}$$

Since this is also the output signal power with maximal ratio combining [12], the performance of the matched filter bound is the same as receive diversity, except for the reduction in gain by M. This reduction in gain by M is due to the fact that for receive diversity, there are M independent sources of noise, whereas with a matched filter, there is only one source of noise. Thus, for a given channel, the degradation in the performance of MLSE as compared to receive diversity is[3]

$$\text{Degradation} = \frac{d_{\min|_{\text{MLSE}}}^2}{d_{\min|_{\text{MFB}}}^2}. \tag{8}$$

[3] It should be emphasized that (8) is the degradation when the receive and transmit channels are the same. Thus, (8) is not the degradation that would be obtained, for a given channel with receive diversity, by replacing receive diversity with transmit diversity since, in general, the channels would differ. However, this performance measure is useful to show why the performance (diversity gain) distribution with transmit diversity is virtually the same as that with receive diversity when considering all possible channels. Also, note that this definition of the degradation is consistent with [10, p. 405]. Note that the degradation is large when the ratio (8) is small, and 10 log {Degradation} becomes more negative as the degradation increases.

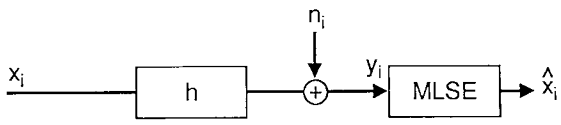

Fig. 2. Discrete time model of transmit diversity.

Since the channel response is a random variable, $d^2_{\min}$ and the degradation are also random variables. Note that with flat Rayleigh fading, the probability distribution of $d^2_{\min|_{\rm MFB}}$, normalized to the mean $d^2_{\min|_{\rm MFB}}$ (averaged over the fading), is given by [12]

$$P(x) = 1 - e^{-x/M} \sum_{k=1}^{M} \frac{(x/M)^{k-1}}{(k-1)!} \tag{9}$$

where

$$x = \frac{d^2_{\min|_{\rm MFB}}}{E[d^2_{\min|_{\rm MFB}}]}. \tag{10}$$

Below, we examine the distribution of the degradation and determine its effect by comparing the distribution of $d^2_{\min|_{\rm MLSE}}$ to that of $d^2_{\min|_{\rm MFB}}$ (9).

III. Results

For $M = 2$, [9] showed that the MLSE receiver can achieve the matched filter bound for any flat fading channel. Thus, transmit diversity with MLSE can have the same diversity gain as receive diversity. For $M = 3$, though, [5] stated that there was some degradation in performance with transmit diversity as compared to receive diversity. Indeed, [9] showed that for $M \geq 3$, there exist real channels (and therefore complex channels as well) for which the matched filter bound, and thus the diversity gain of receive diversity, cannot be achieved. The degradation for the worst real channel is 2.3, 4.2, 5.7, and 7.0 dB for $M = 3,\ 4,\ 5$, and 6, respectively [10]. Since we have complex channels, our worst case channels may have even higher degradation (although none were found). Thus, the worst case degradation grows with M and can exceed the diversity gain, especially at high BER's. However, because the channel is random, these worst case channels, and those channels for which MLSE cannot achieve the matched filter bound, occur with some probability.

To determine the probability distribution of this degradation, we used Monte Carlo simulation. For given M, we generated 10 000 random channels, where each channel consisted of M T-spaced impulses with each impulse having an amplitude that was a randomly generated complex Gaussian number. For each channel, we used the modified program of [11] to determine the minimum Euclidean distance over all possible error sequences and compared this distance to that of the matched filter bound.

Fig. 3 shows the probability distribution of the minimum Euclidean distance (squared) as compared to that of the matched filter bound ($d^2_{\min|_{\rm MFB}}$). Results are shown for $M = 3,\ 4,\ 6,\ 10,\ 20$, and 30. The probability that MLSE cannot achieve the matched filter bound on a given channel is less than 9% for $M = 3$. This probability decreases with M, such that, for $M = 30$ in the simulation, MLSE achieved the matched filter bound in all but one channel out of 10 000.

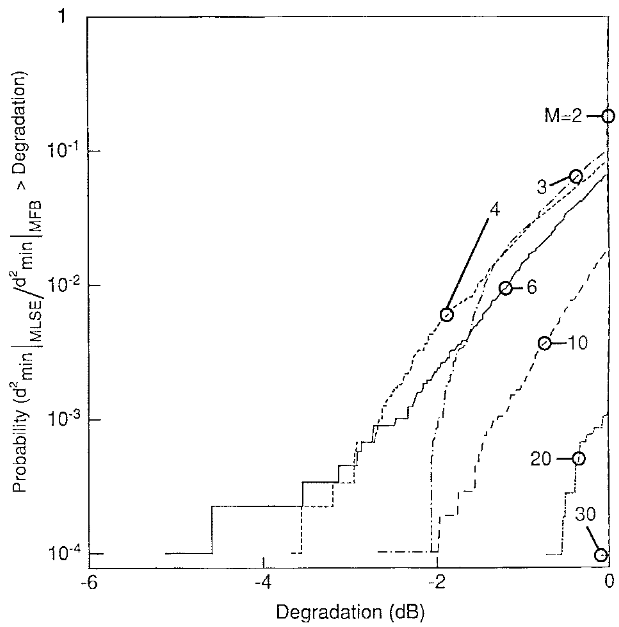

Fig. 3. Probability distribution of the degradation of the diversity gain with MLSE versus the matched filter bound.

For $M = 3$, the worst case degradation is seen to be sharply limited to 2.2 dB, which is close to the 2.3 degradation for the worst real channel [9]. For $M = 4$ and 6, the worst degradation seen with 10 000 random channels was 3.6 and 5.2 dB, respectively, (2.4 and 2.6 dB, respectively, at a 10^{-3} probability), which is significantly less than the worst possible degradation for real channels of 4.2 and 7.0 dB, respectively. As M increases to 10, 20, and 30, the probability of large degradation is shown to decrease (at least for probabilities greater than 10^{-4}). At $M = 30$, only one channel out of the 10 000 random channels had any degradation, and its degradation was only 0.1 dB. Thus, as M increases, although the worst case degradation increases, the probability of worst case degradation decreases.

Next, consider the effect of this degradation on the average BER and the distribution of the BER. Because the degradation has low probability, the effect of the degradation on the average BER is negligible. Thus, in rapidly fading environments, where the average BER is of interest, transmit diversity can achieve the full diversity gain of receive diversity. However, in stationary or slow-fading wireless systems, the effect of the degradation on the distribution of the BER must be considered. The effect of the degradation with MLSE on the probability distribution of the BER depends on the $d^2_{\min|_{\rm MFB}}$ for each channel where the degradation occurs. If this degradation is large only for channels with large $d^2_{\min|_{\rm MFB}}$, then the probability distribution of the BER with MLSE will not be significantly different from that of the BER with the matched filter bound. But if channels with large degradation also have low $d^2_{\min|_{\rm MFB}}$, then the degradation could significantly affect the probability distribution of the BER.

To determine the effect of this degradation on the performance, we used Monte Carlo simulation, as before,

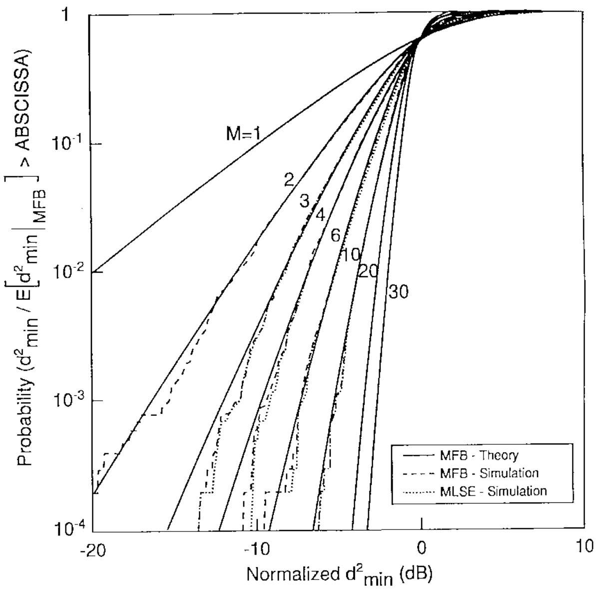

Fig. 4. Probability distribution of the normalized $d^2_{\min}$ with the diversity gain of transmit diversity and MLSE compared to that of receive diversity.

with 10 000 randomly generated channels and compared the probability distribution of the minimum Euclidean distance (squared) of the matched filter bound to that of MLSE. The same channels were used for both the matched filter bound and MLSE.

Fig. 4 shows the probability distribution of $d^2_{\min|_{\mathrm{MFB}}}$ and $d^2_{\min|_{\mathrm{MLSE}}}$ generated by Monte Carlo simulation, along with theoretical results for $d^2_{\min|_{\mathrm{MFB}}}$ (9). Computer simulation results are seen to closely match theoretical results for probabilities down to 10^{-2}. For $M = 2$, simulation results for MLSE are identical to those for the matched filter bound, while for $M \geq 3$, simulation results for MLSE differ by less than 0.1 dB from those for the matched filter bound. These results show that the channels for which MLSE cannot achieve the matched filter bound are generally not the channels with low $d^2_{\min|_{\mathrm{MFB}}}$. Thus, the degradation with MLSE does not significantly affect the probability distribution of the output BER, i.e., transmit diversity with MLSE has within 0.1 dB of the diversity gain of receive diversity, even in stationary environments.

IV. Other Issues

Let us first compare transmit diversity to other techniques that provide diversity at a receiver using multiple transmit antennas only. These techniques include switched diversity with feedback [13] and adaptive retransmission [14]–[17]. With switched diversity with feedback, the transmit antenna is switched when the receiver indicates, using feedback to the transmitter, that the received signal has fallen below a threshold. The advantage of this technique over the transmit diversity technique described in this paper is that the receiver and transmitter are much simpler. However, the disadvantage is that the diversity gain is only that of selection diversity, rather than maximal ratio or optimum combining. This gain is further decreased with processing and propagation delay, which becomes worse with rapid fading. With adaptive retransmission, the multiple-antenna base transmits with the same antenna pattern as that used for reception. The advantages of this technique are that the technique is easy to implement and antenna gain is obtained. However, for the technique to work properly, either the transmit and receive frequencies must be within the coherence bandwidth (which is not true in most wireless systems), or time-division retransmission (different time slots in the same channel are used for receiving and transmitting) must be used. With time-division retransmission, which doubles the data rate in the channel, the time slot must be short enough so that the fading does not change significantly over the time slot, and this is not always possible. For example, in a system with characteristics similar to the North American digital mobile radio standard IS-54 (24.3 k symbols per s with an 81-Hz fading rate), adaptive retransmission with time division is not practical [17].

Transmit diversity also has the advantage that it can be used to obtain diversity gain at multiple remotes (for point-to-multipoint transmission) with a single transmitted signal. The other methods can only be used for diversity gain at one remote.

Transmit diversity is also useful in systems with multiple transmit *and* receive antennas. In this case, the total number of independent fading channels can be $M_T \cdot M_R$ [18], where M_T and M_R are the number of transmit and receive antennas, respectively. Here, transmit diversity can be used with receive diversity to achieve a large $M_T M_R$-fold diversity gain with only a few antennas at the base and remote.

Also, here we have only considered the diversity gain against multipath fading, whereas multiple antennas can be used to suppress interference as well. Indeed, increasing the diversity beyond two or three usually provides little performance improvement against fading, but substantial improvement against cochannel interference [15]–[17], [19]. Interference suppression with fading mitigation using transmit diversity will be studied in a future paper.

However, since transmit diversity with M antennas results in M sources of interference to other users, the interference environment will be different from conventional systems with one transmit antenna. Thus, even if transmit diversity has almost the same performance as receive diversity in noise-limited environments, the performance in interference-limited environments will differ. Also, in our simulations we have considered ideal MLSE with perfect channel estimation. In practice, when increasing the number of antennas, at some point the degradation due to channel estimation error may become greater than the increase in diversity gain. Note also that this channel estimation error may increase with the number of transmit antennas since the transmit power is divided among the transmit antennas (1). In addition, if the channel is dispersive, the diversity gain for the same number of transmit antennas will increase (with ideal MLSE and perfect channel estimation), although the required complexity of the MLSE also increases. Finally, note that for systems with highly elevated base-station antennas, the required antenna separation for uncorrelated channels in the downlink is greater with transmit diversity at the base station than with receive diversity at the remote.

V. Conclusion

In this paper, we studied the diversity gain of transmit diversity with ideal MLSE and an arbitrary number of antennas. We considered BPSK modulation with coherent detection and independent Rayleigh fading between each transmit antenna and receive antenna, with the delay between the transmitted signals such that the received signals are uncorrelated. Using Monte Carlo simulation with Rayleigh fading, we determined the probability distribution of the performance of MLSE. Our results for 2–30 antennas show that transmit diversity can achieve diversity gains within 0.1 dB of receive diversity. Thus, we can obtain the same diversity benefit at the remotes and base stations using multiple base-station antennas only.

Acknowledgment

The author gratefully acknowledges helpful discussions with G. J. Foschini and G. Vannucci and thanks G. Vannucci for providing the programs used in [11].

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Jack H. Winters (S'77–M'81–SM'88–F'96) received the B.S.E.E. degree from the University of Cincinnati, Cincinnati, OH, in 1977 and the M.S. and Ph.D. degrees in electrical engineering from Ohio State University, Columbus, in 1978 and 1981, respectively.

Since 1981, he has been with AT&T Bell Laboratories and is now with AT&T Labs—Research, where he is in the Wireless Systems Research Department. He has studied signal-processing techniques for increasing the capacity and reducing signal distortion in fiber optic, mobile radio, and indoor radio systems and is currently studying adaptive arrays and equalization for indoor and mobile radio.