Optimum Space-Time Processors with Dispersive Interference—Unified Analysis and Required Filter Span

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Abstract -- **In this paper, we consider optimum space-time equalizers with unknown dispersive interference, consisting of a linear equalizer that both spatially and temporally whitens the interference and noise, followed by a decision-feedback equalizer (DFE) or maximum-likelihood sequence estimator (MLSE). We first present a unified analysis of the optimum space-time equalizer, and then show that, for typical fading channels with a given signal-to-noise ratio (SNR), near-optimum performance can be achieved with a finite-length equalizer. Expressions are given for the required filter span as a function of the dispersion length, number of cochannel interferers, number of antennas, and SNR, which are useful in the design of practical, near-optimum space-time equalizers.**

I. Introduction

In wireless communication systems, cochannel interference (CCI) and intersymbol interference (ISI) are major impairments that limit the capacity and data rate. These problems can be mitigated by spatial-temporal (S-T) processing, i.e., temporal equalization with multiple antennas (e.g., [1]).

In typical wireless systems where the cochannel interferers are unknown at the receiver, optimum S-T equalizers, either in a minimum mean square error (MMSE) or maximum signal-to-interference-plus-noise ratio (SINR) sense, consist of a whitening filter, i.e., an equalizer that whitens the cochannel interference both spatially and temporally, followed by a decision-feedback equalizer (DFE) or maximum-likelihood sequence estimator (MLSE) [2].

However, under some channel conditions with dispersive cochannel interference, the whitening filter requires an infinite span to achieve near-optimum performance, even with reasonable signal-to-noise ratios (SNR's). For typical fading channels, though, such channel conditions occur only occasionally, and the required filter span for near-optimum performance is finite in most cases. Since the filter span, specifically both the causal and anticausal portions, determines the required memory of the DFE and MLSE, these filter spans determine the required complexity of near-optimum S-T processors.

In this paper, we first present a unified analysis of the optimum infinite-length S-T processor, considering three receiver types: (i) a MMSE linear equalizer, (ii) a MMSE-DFE, and (iii) a MLSE. The unified analysis includes both previously published results [2]-[5] and additional new material. The objective here is to provide a consistent and comprehensive framework for expressing all these results in a form that is descriptive of the functions and properties of individual filter elements. We then present filter length analyses for all three receivers, by analyzing the z-transform expressions. We show that, with fading channels, the filter spans of these receivers can be truncated such that the average effect of the truncation is small compared to the effect of thermal noise. We then determine the required filter span to achieve near-optimum receiver performance. Using computer simulation, we study the effect of thermal noise on the required filter span for specific fading channels.

II. System Model

We consider a system where $L+1$ cochannel signals are transmitted over independently fading multipath channels to an M-branch diversity receiver. The time-domain complex baseband expression of the received signal on the j-th antenna is

$$r_j(t) = \sum_{i=0}^{L}\sum_{n=-\infty}^{\infty} x_{ni} h_{ij}(t-nT) + n_j(t) \tag{1}$$

where $\{x_{ni}\}$ is the transmitted data sequence from the i-th source, with the *desired* source being indexed by $i=0$, $h_{ij}(t)$ is the overall impulse response of the transmission link between the i-th source and the j-th antenna, T is the symbol period, and $n_j(t)$ is the additive white Gaussian noise at the j-th antenna. The data $\{x_{ni}\}$ are independent, identically distributed (i.i.d.) complex variables with zero mean and unit symbol energy, and are uncorrelated between sources.

The frequency-domain expression of the above received signal is

$$R_j(f) = \sum_{i=0}^{L} X_i(f) H_{ij}(f) + N_j(f) \tag{2}$$

where $R_j(f)$, $X_i(f)$, $H_{ij}(f)$, and $N_j(f)$ are the Fourier Transforms of $r_j(t)$, $\{x_{ni}\}$, $h_{ij}(t)$, and $n_j(t)$, respectively. Since the data have unit symbol energy, $E[|X_i(f)|^2] = 1$ for $|f| \le 1/(2T)$, where $E[\cdot]$ denotes expectation. The noise at each antenna has two-sided power spectrum density N_0.

The general space-time receiver using a DFE is shown in Fig. 1 (a LE receiver model can be obtained by setting the feedback filter response to zero). It consists of a linear feedforward filter $W_j(f)$, $j = 0, 1, \ldots, M-1$, on each branch, a combiner, symbol-rate sampler, slicer, and synchronous linear feedback filter $B(f)$. The feedforward filters $\{W_j(f)\}$ are shown as continuous-time filters, but they can be implemented in practice using fractionally-spaced tapped delay lines. The input to the feedback filter is the decided data $\{\hat{x}_{n0}\}$ for the desired source. We assume correct decisions ($\hat{x}_{n0} = x_{n0}$) throughout this study.

The input to the slicer (i.e., the space-time processor output) is denoted by sequence $\{y_n\}$, with its Fourier transform $Y(f)$ given by

$$Y(f) = \sum_{j=0}^{M-1}\sum_{m=-\infty}^{\infty} W_j\left(f-\frac{m}{T}\right) R_j\left(f-\frac{m}{T}\right) - B(f)X_0(f) \tag{3}$$

The summation with respect to m in the above equation is a result of spectrum folding due to symbol-rate sampling.

Based on the MMSE criterion, the filters are optimized by minimizing the mean square error (MSE):

$$\varepsilon = E[|y_n - x_{n0}|^2] = T\int_{-\frac{1}{2T}}^{\frac{1}{2T}} E[|Y(f) - X_0(f)|^2]\, df \tag{4}$$

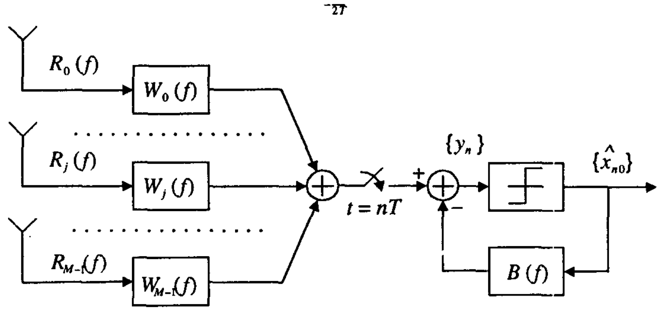

Fig. 1. A space-time DFE receiver

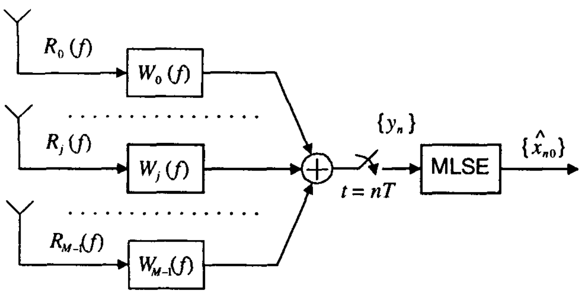

Fig. 2. A space-time MLSE receiver

Fig. 2 shows a space-time receiver using a MLSE. Here, the goal of optimization is to maximize the signal power (without suppressing ISI) to CCI plus noise power ratio, while whitening the CCI and noise components of the input $\{ y_n \}$ to the MLSE.

III. Unified Infinite-Length Theory

A. Optimum Filter Expressions for DFE and LE Receivers

The MMSE solution for the feedforward filters $\{ W_j(f) \}$ with unconstrained length can be derived by using (2)-(4) and setting the derivatives $\left\{ \frac{\partial \varepsilon}{\partial W_j\left(f - \frac{m}{T}\right)} \right\}$ to zero. This yields

$$W = [R_S + R_{I+N}]^{-1} H_0^* (1 + B(f)) \tag{5}$$

where

$$W \triangleq \left[W_0\left(f-\frac{J}{T}\right) \dots W_{M-1}\left(f-\frac{J}{T}\right) \dots \dots W_0\left(f+\frac{J}{T}\right) \dots W_{M-1}\left(f+\frac{J}{T}\right) \right]^T$$

$$H_i \triangleq \left[H_{i0}\left(f-\frac{J}{T}\right) \dots H_{i,M-1}\left(f-\frac{J}{T}\right) \dots \dots H_{i0}\left(f+\frac{J}{T}\right) \dots H_{i,M-1}\left(f+\frac{J}{T}\right) \right]^T$$

$$R_S \triangleq H_0^* H_0^T$$

$$R_{I+N} \triangleq \sum_{i=1}^{L} H_i^* H_i^T + N_0 I$$

R_S is the correlation matrix of the desired signal, R_{I+N} is the correlation matrix of the interference plus noise, I is the identity matrix, and the superscripts * and T denote complex conjugate and transpose, respectively. We assume that the desired and CCI sources are strictly bandlimited to $f = \pm J'/(2T)$ (J' is a positive integer), and therefore $J = (J'-1)/2$ when J' is odd, and $J = J'/2$ when J' is even (e.g., $J' = 1$ and $J = 0$ when there is no excess bandwidth).

Using the matrix inversion lemma, we can rewrite (5) as [10]

$$W = R_{I+N}^{-1} H_0^* \frac{1 + B(f)}{1 + \Gamma(f)} \tag{6}$$

where

$$\Gamma(f) = \frac{W^\dagger R_S W}{W^\dagger R_{I+N} W} \tag{7}$$

is the signal-to-interference-plus-noise power density ratio at frequency f. Substituting (6) into (7) yields

$$\Gamma(f) = H_0^T R_{I+N}^{-1} H_0^* \tag{8}$$

Equation (6) gives the form of the MMSE solution well known in array processing [2] (except for the consideration of spectrum folding and feedback filtering). This equation indicates that the optimum feedforward filter consists of a space-time filter $R_{I+N}^{-1} H_0^*$, which performs spatial prewhitening of CCI and noise and matching to the desired channel, followed by a temporal filter $\frac{1 + B(f)}{1 + \Gamma(f)}$.

The optimum feedback filter $B(f)$ can be determined through spectrum factorization. Substituting (6) into (3), and using (4), we obtain

$$\varepsilon = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{|1 + B(f)|^2}{1 + \Gamma(f)} df \tag{9}$$

$$= \frac{1}{2\pi \mathrm{j}} \oint \frac{(1 + B(z))\,(1 + B^*(z^{-1}))}{1 + \Gamma(z)} \frac{dz}{z} \tag{10}$$

where $B(z)$ and $\Gamma(z)$ are the z-transform equivalents of $B(f)$ and $\Gamma(f)$, $\mathrm{j} = \sqrt{-1}$, and The contour of the integration in (10) is the unit circle. Using spectral factorization theory [5], $1 + \Gamma(f)$ and $1 + \Gamma(z)$ can be written as

$$1 + \Gamma(f) = S_0 |G(f)|^2 \ ; \ \ 1 + \Gamma(z) = S_0 G(z) G^*(z^{-1}) \tag{11}$$

where the constant S_0 is given by

$$S_0 = e^{\langle \ln(1 + \Gamma(f)) \rangle} \tag{12}$$

and

$$\langle \cdot \rangle \triangleq T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} [\cdot]\, df \tag{13}$$

and $G(z)$ is *canonical*, meaning that it is *causal* ($g_k = 0$ for $k < 0$), *monic* ($g_0 = 1$), and *minimum-phase* (all of its poles are inside the unit circle, and all of its zeroes are on or inside the unit circle).

Using the Schwarz inequality, it can be shown that the MSE in (9) is minimized when

$$1 + B(f) = G(f) \ ; \ \ 1 + B(z) = G(z) \tag{14}$$

Substituting (11)-(14) into (6) and (9), we obtain

$$W_{\text{DFE}} = R_{I+N}^{-1} H_0^* \frac{1}{S_0 G^*(f)} \tag{15}$$

and

$$\varepsilon_{\text{DFE}} = \frac{1}{S_0} = e^{-\langle \ln(1 + \Gamma(f)) \rangle} \tag{16}$$

As will be relevant later, we can write $1 + B(z)$ also as

$$1 + B(z) = C[1 + \Gamma(z)] \tag{17}$$

where $C[\cdot]$ denotes canonical factor. The corresponding Fourier transform is given as

$$1 + B(f) = C[1 + \Gamma(f)] \tag{18}$$

Using the above expression, we can write (6) as

$$W_{\text{DFE}} = R_{I+N}^{-1} H_0^* \frac{C[1 + \Gamma(f)]}{1 + \Gamma(f)} \tag{19}$$

The optimum LE is obtained by setting $B(f)$ to zero in (6) and (9):

$$W_{\text{LE}} = R_{I+N}^{-1} H_0^* \frac{1}{1 + \Gamma(f)} \tag{20}$$

and

$$\varepsilon_{\text{LE}} = \left\langle \frac{1}{1 + \Gamma(f)} \right\rangle \tag{21}$$

B. An Alternative Solution for DFE and LE Receivers

The optimum space-time filter solution in (6) is based on a general model which does not make any prior assumptions regarding the filter structure. Without loss of optimality, an analytical receiver model suggested by many in the literature (e.g., [3][4][6][7]) assumes the use of a bank of matched filters $\{ H_{ij}^*(f) \}$, each corresponding to the signal source i on diversity branch j, which, after diversity combining, is followed by a bank of T-spaced transversal filters $\{ V_i(f) \}$, each corresponding to the signal source i (see Fig. 3). This analytical model leads to a different form of solutions which are important to our filter length analysis. The following derivation is similar to the LE receiver derivation in [4], but here we also provide the solution for the DFE.

In Fig. 3, the Fourier transform of the input to the slicer is

$$Y(f) = \sum_{i=0}^{L} D_i(f) X_i(f) + \aleph(f) - B(f) X_0(f) \tag{22}$$

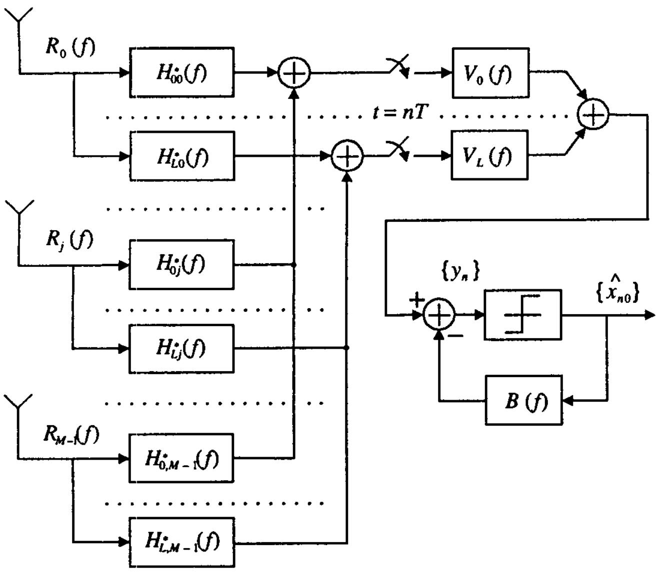

Fig. 3. An equivalent model of the space-time DFE receiver in Fig. 1.

where $D_0(f)$ and $D_i(f)$ are the overall channel and feedforward filter responses for the desired signal and the i-th interference, and $\aleph(f)$ is the noise at the combined output of the feedforward filters. Let $D = [D_0(f) \ \ldots \ D_L(f)]^T$ and $V = [V_0(f) \ \ldots \ V_L(f)]^T$. We then have the following relationship:

$$D = PV \tag{23}$$

where P is an $(L+1) \times (L+1)$ correlation matrix whose (a, b)-th element p_{ab} is given by

$$p_{ab} = \sum_{j=0}^{M-1} \sum_{m=-\infty}^{\infty} H_{aj}\left(f - \frac{m}{T}\right) H^*_{bj}\left(f - \frac{m}{T}\right). \tag{24}$$

The MSE for this receiver is given by [10]

$$\varepsilon = \langle V^\dagger P^\dagger P V + N_0 V^\dagger P^\dagger V + |1 + B(f)|^2 - 2\mathrm{Re}[(1 + B^*(f))\, V^T P^T U] \rangle \tag{25}$$

where $U = [1, 0, \ldots, 0]^T$ is a column vector with $L+1$ rows.

The MMSE solution for V is obtained by solving $\dfrac{\partial \varepsilon}{\partial V_i(f)} = 0$ for $i = 0$ to L. We then obtain

$$V = (P + N_0 I)^{-1} U (1 + B(f)) \tag{26}$$

It can be shown that this receiver achieves the same MMSE as (16) (or (21) in the case of a LE receiver), and that

$$1 + \Gamma(f) = \frac{1}{N_0} \frac{1}{U^T (P + N_0 I)^{-1} U} \tag{27}$$

Again, $1 + B(z)$ is the canonical factor of $1 + \Gamma(z)$. Accordingly, we can rewrite (26) as

$$V_{\mathrm{DFE}} = (P + N_0 I)^{-1} U \, C[1 + \Gamma(f)] \tag{28}$$

For a LE receiver

$$V_{\mathrm{LE}} = (P + N_0 I)^{-1} U \tag{29}$$

C. Optimum Linear Filtering for MLSE

Fig. 4 shows an equivalent model of the MLSE receiver in Fig. 2. The front-end filters are now represented by spatial filters $\{W_j'(f)\}$, which maximize SINR of their combined output, followed by a post-whitening filter $\Psi(f)$. Let W' denote the vector of $\{W_j'(f)\}$. The signal-to-interference-plus-noise power density ratio $\Gamma(f)$ is then (cf., (7))

$$\Gamma(f) = \frac{W'^\dagger R_S W'}{W'^\dagger R_{I+N} W'} \tag{30}$$

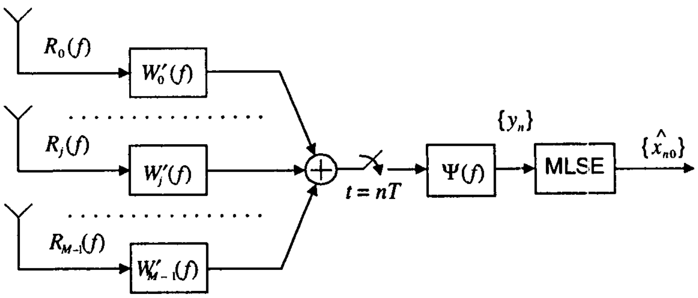

Fig. 4. An equivalent model of the space-time MLSE receiver in Fig. 2

The optimum W' is obtained by solving $\dfrac{\partial \Gamma(f)}{\partial W_j'(f)} = 0$, for $j = 0, 1, \ldots, M-1$; this gives the relationship

$$R_S W' = \Gamma(f) R_{I+N} W' \tag{31}$$

The maximum $\Gamma(f)$ is therefore given by the maximum eigenvalue of $R_{I+N}^{-1} R_S$. Let W'_{opt} be the eigenvector corresponding to this maximum eigenvalue, and substitute $W' = W'_{\mathrm{opt}}$ into (30). We obtain

$$W'_{\mathrm{opt}} = \beta(f) R_{I+N}^{-1} H_0^* \tag{32}$$

where

$$\beta(f) = \frac{W'^\dagger_{\mathrm{opt}} R_{I+N} W'_{\mathrm{opt}}}{W'^\dagger_{\mathrm{opt}} H_0^*} \tag{33}$$

Equation (32) has the same form as (6). As a result, we obtain the same expression for $\Gamma(f)$ as (8). By factoring $\Gamma(f)$ as

$$\Gamma(f) = S_1 |C[\Gamma(f)]|^2 \tag{34}$$

we can find a post-whitening filter

$$\Psi(f) = \frac{C[\Gamma(f)]}{\beta(f)\Gamma(f)} \tag{35}$$

which satisfies

$$W^\dagger R_{I+N} W = |\beta(f)|^2 |\Psi(f)|^2 H_0^T R_{I+N}^{-1} H_0^* \tag{36}$$

$$= \frac{|C[\Gamma(f)]|^2 \Gamma(f)}{|\Gamma(f)|^2} = \frac{1}{S_1} = \text{constant}$$

where the overall filter response W is given by

$$W = W' \Psi(f) = R_{I+N}^{-1} H_0^* \frac{C[\Gamma(f)]}{\Gamma(f)} \tag{37}$$

Comparing (37) to (19), we find that

$$W_{\mathrm{MLSE}} = W_{\mathrm{DFE}} \cdot \frac{1 + \Gamma(f)}{C[1 + \Gamma(f)]} \cdot \frac{C[\Gamma(f)]}{\Gamma(f)} \tag{38}$$

This relationship is extendable to the case where we use the analytical feedforward filter model in Fig. 3 to represent the front-end filter of the MLSE receiver. Thus, we can also write

$$V_{\mathrm{MLSE}} = V_{\mathrm{DFE}} \cdot \frac{1 + \Gamma(f)}{C[1 + \Gamma(f)]} \cdot \frac{C[\Gamma(f)]}{\Gamma(f)} \tag{39}$$

IV. Filter Length Analysis

Our filter length analysis is based on counting the number of zeroes and poles in the z-transform expression of the optimum space-time filter. For all the three receivers (LE, DFE, and MLSE), there are two forms of optimum filter solutions: (i) one based on the general model (with a linear filter on each branch), and (ii) one based on the analytical model (with a bank of matched filters on each branch, followed by common filters). The filter length determined by each solution is valid under different assumptions: The filter length based on (i) is valid when $M(2N+1) < L+1$, i.e., when the number of interferers is equal to or greater than the order of diversity due to multiple antennas and excess bandwidth. The filter length based on (ii) is valid when

$M(2N+1) \geq L+1$. The reason for these different conditions will become apparent later.

Since the general analytical approach for determining the filter length is the same for both solution forms, we only provide details for one of them below. We choose to work on case (ii) because it is slightly more complicated than the other case, and because the condition under which it is valid (the order of diversity exceeding the number of dominant interferers) is where the most interference suppression is achieved, i.e., an array with M antennas can null up to $M-1$ interferers [9].

We begin by working on the MMSE solution for the DFE receiver. The z-transform equivalent of (26) is

$$V = (P+N_0 I)^{-1} U(1+B(z)) \stackrel{\Delta}{=} Q^{-1} U(1+B(z)) \tag{40}$$

where $V \stackrel{\Delta}{=} \left[V_0(z) \ \ldots \ V_L(z)\right]^T$ and $Q \stackrel{\Delta}{=} P+N_0 I$. The element p_{ab} of matrix P is the z-transform of the sampled sequence of

$$\sum_{j=0}^{M-1} \int_{-\infty}^{\infty} h_{aj}(\tau)\, h^*_{bj}(\tau - t)\, d\tau \tag{41}$$

Thus, each element q_{ab} of matrix Q has a two-sided response such that it includes both a causal factor and an anticausal factor of equal length. If we assume that all channels $\{h_{ij}(t)\}$ have a finite memory of K symbol periods ($h_{ij}(t) = 0$ for $t<0$ and $t>KT$), then the causal and anticausal factors of q_{ab} will be polynomials of order K.

Using the matrix identity [8]

$$Q^{-1} = \frac{Q_{\mathrm{adj}}}{|Q|} \tag{42}$$

where Q_{adj} is called the *adjugate matrix* of matrix Q, and the (a, b)-th element Q_{ab} of the transpose matrix of Q_{adj} is called the *cofactor* corresponding to the (a, b)-th element q_{ab} of matrix Q, we can rewrite (40) as

$$\begin{aligned} V &= \frac{Q_{\mathrm{adj}}}{|Q|} U(1+B(z)) \\ &= \frac{\left[Q_{00}\ Q_{01}\ \ldots\ Q_{0L}\right]^T}{|Q|}(1+B(z)) \end{aligned} \tag{43}$$

$1+B(z)$ is given by (see (27))

$$1+B(z) = C[1+\Gamma(z)] = C\left[\frac{1}{N_0 U^T Q^{-1} U}\right] = \frac{C[|Q|]}{C[Q_{00}]} \tag{44}$$

Thus, we obtain

$$V = \frac{\left[Q_{00}\ Q_{01}\ \ldots\ Q_{0L}\right]^T}{(|Q|/C[|Q|]) \cdot C[Q_{00}]} \tag{45}$$

We now focus on each term in (45). Defining a permutation σ as a one-to-one mapping σ: (0, 1, ..., L) $\rightarrow$ (σ_0, σ_1, ..., σ_L), the determinant of Q is [8]

$$|Q| = \sum_{\sigma} \mathrm{sgn}(\sigma)\, q_{0\sigma_0} q_{1\sigma_1} \cdots q_{L\sigma_L} \tag{46}$$

where $\mathrm{sgn}(\sigma) = +1$ or -1 depending on whether the number of exchanges in permutation σ is even or odd, and the summation is taken over all $(L+1)!$ permutations σ. Note that the product of two polynomials of order a and b results in a polynomial of order $a+b$, while the sum of them gives a polynomial of order max[a, b]. Since each element q_{ab} of matrix Q includes a causal factor and an anticausal factor, each of order K, $|Q|$ will, in general, have a causal factor and anticausal factor, each of order $K(L+1)$. Similarly, Q_{ab} will, in general, have a causal factor and anticausal factor, each of order KL. Accordingly, $|Q|/C[|Q|]$ will be anticausal (and maximum-phase) with order $K(L+1)$, and $C[Q_{00}]$ will be causal (and minimum-phase) with order KL. Combining these results together, the causal part of each filter $V_i(z)$ (for $i>0$) will have KL zeros and KL poles, and its anticausal part will have KL zeros and $K(L+1)$ poles. Since each front-end matched filter $h^*_{ij}(-t)$ is anticausal with length K (we can always set the synchronization timing such that $h_{ij}(t)$ is a causal function), the overall feedforward filter on each branch will have a causal part with KL zeros and KL poles, and an anticausal part with $K(L+1)$ zeroes and $K(L+1)$ poles.

In general, a pole filter has an infinite impulse response. Nevertheless, we can always truncate a pole filter which is causal and minimum-phase, or anticausal and maximum-phase, such that the effect of truncation is small compared to the background noise. Thus, the lengths in units of T of the causal and anticausal parts (denoted as C and A, respectively) of the optimum feedforward filter can be given as

DFE ($M(2N+1) \geq L+1$):

$$\begin{aligned} C &= KL(1+\alpha) \\ A &= K(L+1)(1+\alpha) \end{aligned} \tag{47}$$

Here, α determines the truncated length of a pole filter $(1-\xi z)^{-1}$ or $(1-\xi z^{-1})^{-1}$. Note that $C=0$ when $L=0$; thus, in the absence of CCI, the optimum feedforward filter of a DFE is anticausal.

Note that the required length of the feedback filter is $K+C$, since the optimum feedback filter completely cancels the postcursors of the desired signal.

Similarly, we can estimate the filter length for a LE receiver using (43), with $B(z)$ set to zero; this gives

LE ($M(2N+1) \geq L+1$):

$$\begin{aligned} C &= K(L+1)(1+\alpha) - K \\ A &= K(L+1)(1+\alpha) \end{aligned} \tag{48}$$

The causal length of the LE receiver is greater (by $K\alpha$) than that of the feedforward filter of the DFE receiver.

The above results are valid under the condition that the MMSE solution in (43) is *compact*, meaning that there is no cancellation of highest-order terms in the summation in (46) for all determinants. By working on specific examples, we found (43) to be compact when $M(2N+1) \geq L+1$. Otherwise, the MMSE solution based on the general filter model (given in (6)) is compact.

Using the same analytical approach as above, we estimate the filter length for the case $M(2N+1) < L+1$ as

LE and DFE ($M(2N+1) < L+1$):

$$\begin{aligned} C &= KM(2N+1)(1+\alpha) - K \\ A &= KM(2N+1)(1+\alpha) \end{aligned} \tag{49}$$

The filter length results are the same for both LE and DFE receivers in this case.

We now focus on the MLSE receiver. Using (45) and the relationship in (39), we obtain

$$V_{\mathrm{MLSE}} = \frac{\left[Q_{00}\ Q_{01}\ \ldots\ Q_{0L}\right]^T}{((|Q| - N_0 Q_{00})/C[|Q| - N_0 Q_{00}]) \cdot C[Q_{00}]} \tag{50}$$

Comparing (50) with (45), we see that individual terms in the two equations have the same highest order and, thus, the two filters have the same length. This is also true when we compare the filter lengths using the general filter model. We therefore conclude that the optimum front-end filter for the MLSE receiver has the same length as the optimum feedforward filter of the DFE receiver; namely

MLSE ($M(2N+1) \geq L+1$):

$$\begin{aligned} C &= KL(1+\alpha) \\ A &= K(L+1)(1+\alpha) \end{aligned} \tag{51}$$

and

MLSE ($M(2N+1) < L+1$):

$$\begin{aligned} C &= KM(2N+1)(1+\alpha) - K \\ A &= KM(2N+1)(1+\alpha) \end{aligned} \tag{52}$$

Note that the above analysis does not take into account the effect of thermal noise, i.e., strictly speaking, the filter length expressions given above are valid only when the input SNR approaches infinity. As the SNR decreases, we expect the required length of the whitening filter V or $R^{-1}_{I+N} S(z)$ ($S(z)$ denotes the temporal filter, e.g., $S(z) = 1/(1+\Gamma(z))$ for a LE receiver) to decrease, and eventually approach zero when thermal noise dominates both CCI and ISI. Although analytical results are not available, it is possible to study this effect through numerical examples, as shown in the next section. Nevertheless, the general relationship regarding how the required length of the whitening filter varies with the dispersion length, number of interferers, and order of diversity, should remain unchanged for any given SNR.

V. Numerical Results

We now study the effect of thermal noise on the required filter span of DFE and LE receivers. As discussed earlier, we expect the required filter span of a MLSE receiver to be the same as that of a DFE receiver (although this needs to be proven for all given SNR's). For the purpose of illustration, we assume a single-carrier system using quaternary phase shift keying (QPSK) with Nyquist filtering. We also assume a *multi-ray* delay profile for all the channels, where all the rays are of equal power, independently Rayleigh faded, and uniformly spaced by an interval T (the symbol period). The fading is assumed to be independent for different signal sources and diversity branches. We only consider the case of $M \geq L+1$, i.e., the receiver has a sufficient number of antennas to suppress all dominant interferers (the remaining interference can be treated as Gaussian noise if its total energy is sufficiently low). The performance is given in terms of the average bit error rate (BER) over Rayleigh fading, where the fading of individual channels was generated by Monte-Carlo simulation.

In Figs. 5 to 8, we plot the average BER as a function of the length (the number of symbol-spaced taps) of the causal and/or anticausal portion of the filter on each diversity branch. The total filter length is $C+A+1$, where C and A are the length of the causal and anticausal portions, respectively, as defined earlier. In all the figures, the BER is shown to decrease with the filter length until it reaches an asymptotic value (a "floor"). The arrows on the right side of each curve show the corresponding infinite-length BER (due to an artifact of the computation, the infinite-length BER's do not necessarily match the BER floors exactly). The triangle symbol on each curve indicates the required filter length to achieve "near-optimum" performance, where "near-optimum" is defined here as being within 5% of the BER floor. We assume in all the DFE results (Figs. 5 to 7) that the feedback filter is sufficiently long such that it completely cancels the postcursor ISI of the desired signal.

Fig. 5 shows the effect of the average input SNR $\bar{\gamma}$ (the average is over Rayleigh fading) on the required filter length of the DFE receiver, assuming a 4-ray channel model ($K=3$) with no CCI ($L=0$). In this case, the optimum feedforward filter is anticausal (see (47)), so the performance is given only as a function of A. The results for $M=1$ show that the required filter length to achieve near-optimum performance increases by one for every 3 dB increase in SNR in most cases (the required length stays unchanged when $\bar{\gamma}$ increases from 15 dB to 18 dB). We will discuss this relationship in more detail later. The example results for $M=2$ and $M=4$ show that the required filter length does not change with the number of diversity antennas.

Fig. 6 shows results with different dispersion lengths, assuming $L=1$, $M=2$, and $\bar{\gamma}=18$ dB. The performance is given as a function of the lengths of both the causal and anticausal portions of the filter; the results for each portion are obtained by assuming a sufficient length for the other portion. These results show an approximately proportional relationship between the required filter length and the channel dispersion length:

$$\begin{aligned} C &\approx 1.8K \\ A &\approx 4.6K \end{aligned} \qquad \text{for } L=1,\ M=2, \text{ and } \bar{\gamma}=18 \text{ dB} \tag{53}$$

Fig. 7 shows results for different values of L, assuming a 4-ray channel model ($K=3$) with $M=4$ and $\bar{\gamma}=9$ dB. The average SIR is 0 dB, -3 dB, and -4.8 dB for $L=1$, $L=2$, and $L=3$, respectively. The results show that the required filter length grows linearly with the number of interferers.

So far, all the simulation results generally agree with the analytical results in (47), i.e., the required filter length to achieve near-optimum performance grows linearly with K and L, but it does not depend on M. In addition, we found in this section that the required filter length grows almost linearly with the average input SNR in dB. Combining these results together, and taking into account the fact that the anticausal part of the filter always includes a matched filter of length K, we obtain the following empirical formulae for the required filter length:

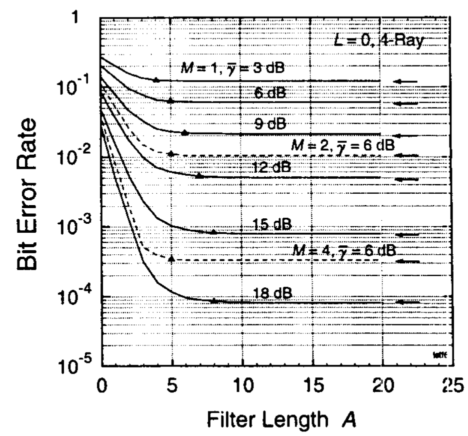

Fig. 5. Effect of SNR $\bar{\gamma}$ on the required filter length of a space-time MMSE-DFE. $L=0$. 4-ray channel ($K=3$)

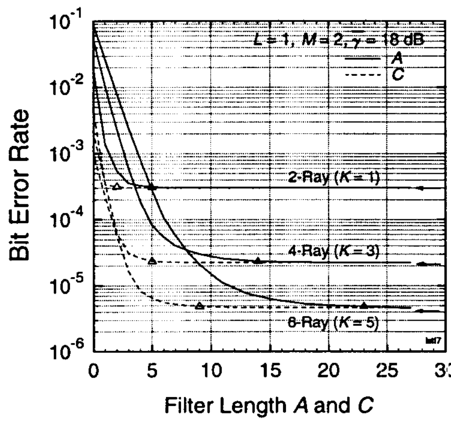

Fig. 6. Effect of channel dispersion length K on the required filter length of a space-time MMSE-DFE. $L=1$, $M=2$, and $\bar{\gamma}=18$ dB

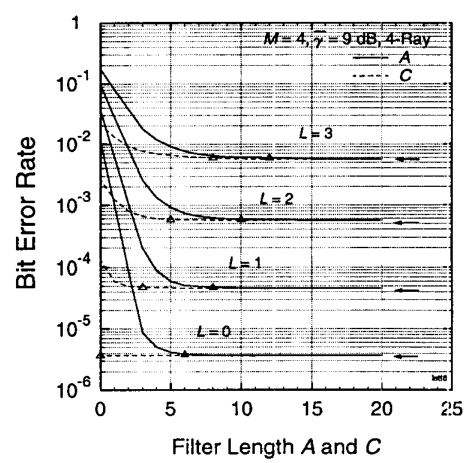

Fig. 7. Effect of the number of interferers L on the required filter length of a space-time MMSE-DFE. $M=4$, $K=3$, and $\bar{\gamma}=9$ dB

DFE:

$$C \approx KL\phi(\bar{\gamma}) \quad (54)$$
$$A \approx K + K(L+1)\phi(\bar{\gamma})$$

where

$$\phi(\bar{\gamma}) = \frac{\bar{\gamma}}{10} \quad (55)$$

and $\bar{\gamma}$ is in dB. The good agreement between the filter lengths predicted by (54) and the simulation results are shown in Tables I to III. Although not shown here, we also found good agreement when testing the empirical formulae against simulation results withother sets of parameter values.

TABLE I:
REQUIRED FILTER LENGTH RESULTS IN FIG. 5, COMPARED WITH PREDICTED LENGTHS BASED ON (54) (SHOWN IN PARENTHESES). $L = 0, M = 4$, AND $K = 3$.

$\bar{\gamma}$ in dB	3	6	9	12	15	18
A	4 (3.9)	5 (4.8)	6 (5.7)	7 (6.6)	8 (7.5)	8 (8.4)

TABLE II:
REQUIRED FILTER LENGTH RESULTS IN FIG. 6, COMPARED WITH PREDICTED LENGTHS BASED ON (54) (SHOWN IN PARENTHESES). $L = 1, M = 2$, AND $\bar{\gamma} = 18$ dB.

	$K = 1$	$K = 3$	$K = 5$
C	2 (1.8)	5 (5.4)	9 (9)
A	5 (4.6)	14 (13.8)	23 (23)

TABLE III:
REQUIRED FILTER LENGTH RESULTS IN FIG. 6, COMPARED WITH PREDICTED LENGTHS BASED ON (54) (SHOWN IN PARENTHESES). $L = 1, M = 2$, AND $\bar{\gamma} = 18$ dB.

	$K = 1$	$K = 3$	$K = 5$
C	2 (1.8)	5 (5.4)	9 (9)
A	5 (4.6)	14 (13.8)	23 (23)

Despite its empirical nature, (54) has meaningful analytical justifications: First, it gives the same form of expression for C as the analytical result in (47), except for the dependence on the SNR (which is also expected of α in (47)). Second, when $\gamma \to \infty$ such that $\phi(\gamma) >> 1$, the expression for A in (54) becomes $A \to K(L+1)\phi(\bar{\gamma})$; thus, we also obtain the same form of expression for A as (47). As discussed earlier, (47) is also valid when $\bar{\gamma} \to \infty$. Finally, (54) gives the length A as the sum of the matched filter length K and the length of the anticausal portion of the whitening filter $K(L+1)\phi(\bar{\gamma})$ which decreases with decreasing input SNR; this agrees with the intuition that the length of the whitening filter should approach zero when thermal noise dominates both CCI and ISI.

As for the LE receiver, the analytical results in (48) show that it has the same anticausal length A as that of the DFE receiver, and its causal length is given by $C = A - K$. Thus, we simply modify (54) as

LE:

$$C \approx K(L+1)\phi(\bar{\gamma}) \quad (56)$$
$$A \approx K + K(L+1)\phi(\bar{\gamma})$$

The required filter length results in Fig. 8 agree well with the lengths predicted by the above empirical expressions.

Equations (54) and (56) give useful empirical expressions for predicting the required filter span of space-time DFE and LE receivers for a given SNR. Although the empirical function $\phi(\bar{\gamma})$ is given in (55) only for a specific channel model (Rayleigh fading and a uniform delay spread profile), this function can be easily determined for other channel environments, by studying only the single-antenna, no CCI performance, similar to the way we determined $\phi(\bar{\gamma})$ from the results in

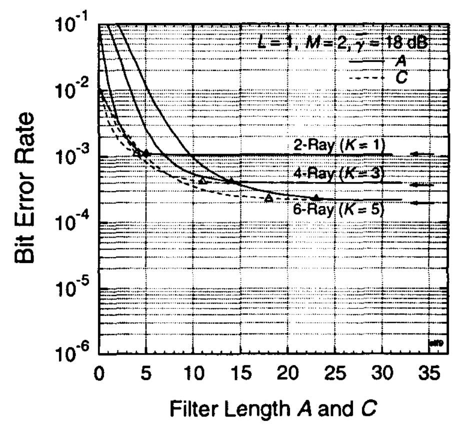

Fig. 8. Required filter length for space-time MMSE-LE. $L = 1$, $M = 2$, and $\bar{\gamma} = 18$ dB

Fig. 5 and Table I.

VI. CONCLUSION

In this paper, we studied optimum space-time equalization of dispersive fading channels with cochannel interference. We first presented a unified analysis of optimum space-time equalizers, consisting of a linear filter on each antenna branch, followed by a DFE or MLSE. In this analysis, we derived explicit expressions for the linear filter (e.g., (19), (28), and (38)), which are novel to the best of our knowledge. Using z-transform analysis, we also derived expressions for the linear filter length, showing that the required span is proportional to the channel dispersion length and the number of interferers. We then used computer simulation to derive empirical expressions for the required filter span which show that the span is also proportional to the input SNR in dB. The derived empirical expressions for the required span are in good agreement with simulation results with Rayleigh fading and a uniform delay spread profile. These expressions are useful in the design of practical, near-optimum space-time equalizers.

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