SINUSOIDAL MODELING AND PREDICTION OF FAST FADING PROCESSES

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Abstract

Fast fading is a major difficulty for wireless systems operating at higher and higher carrier frequencies. As suggested by the Jakes fading simulation model, fading processes due to multipath propagation can be, by nature, modeled in baseband as a combination of multiple complex superimposed sinusoids. In this paper, starting with the observation of a flat faded and noise corrupted baseband modulated signal , we apply and modify the ROOT-MUSIC algorithm to model and further predict the complex fading process. The modeling/prediction performance is evaluated by both simulated and real-world fading data. It is shown that short-term prediction is feasible, and we conclude that the proposed approach may find many applications to improve system performance under fast fading situations.

1. Introduction

It is well known that a principal detrimental factor in limiting the performance of wireless mobile communication systems is signal fading, in that it is unknown and imposes multiplicative distortion on the transmitted signal. An effective technique to tackle this difficulty is spatial diversity which uses multiple receiving antennas and employs a certain scheme to combine the multiple independently-faded received signals. However, for such a scheme to be successful, it is often necessary to estimate or track the fading process to determine the appropriate combiner weights [1]. Under slow fading conditions, fading estimation is often done by a windowing and averaging technique. However, because the fading rate is proportional to the RF frequency, the fading becomes faster as the carrier frequency is increased. For such a fast varying fading channel, the conventional averaging method needs to use a shorter window length, resulting in degraded performance. Furthermore, the estimated fading process obtained by the conventional windowing method also suffers from a time lag. Without introducing a decision delay of half the window size, the lag will also lead to some performance degradation. Aware of the above problems, we thus attempt to study the problem of fading modeling for the purpose of fading prediction, which means the prediction of the future fading process (envelope and phase) based on noisy observations of the faded modulated signal.

The problem of fading prediction has been studied by [2] who assumed that the fading process could be modeled by a small number of sinusoids. In investigating the statistical properties of fading, researchers often began with a model of multiple superimposed sinusoids which correspond to many reflected rays with different Doppler frequency offsets, attenuations, and phases. This physical conception leads to a sinusoidal model with its parameters regarded as random variables. Based on this idea, Jakes proposed a widely adopted Rayleigh fading model to generate artificial fading process for system simulations [3]. In fact, in both LOS and non-LOS environments, spectral analysis of real-world fading data strongly supports the conjecture that the complex baseband fading process mainly consists of a small number of sinusoids. However, we are unaware of any studies of fading prediction using real-world data generated using rapidly-moving mobiles. All of the above facts motivate us to this study. In modeling the fading as a superimposed sinusoidal process, it may be reasonable to assume that the sinusoidal parameters can be treated as unknown constants during a short enough interval. Thus, we speculate that the fading process is in fact a *deterministic* sinusoidal process with time-varying parameters. With such view, we consider the problem of fading modeling and subsequent fading prediction. It is easy to see that if the receiver is equipped with such a capability, then significant performance improvement may be possible [2].

Now we should point out that Makov-type models have also long been used for studying the statistical properties of fading processes [3,4,5], such as level crossing rate, etc. For such a model to be applied to fading prediction, one naturally first consider the Kalman filtering techniques [1, Ch.14]. However, since the whole state-space signal model entails an

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unpredictable process noise and a measurement noise, it is thus feasible to predict only very few steps ahead. Besides, this model has no direct link to the physical origin that gives rise to the fading phenomenon. So we consider it not appropriate for fading prediction.

The paper is organized as follows. In Sec. 2 we state the signal model and problem formulation. In Sec. 3, the sinusoidal modeling for fading process is presented by applying the ROOT-MUSIC frequency estimation method [6]. In Sec.4, the fading prediction problem, performance measures, and schemes for improving the prediction performance are proposed. Then the method is evaluated by using both computer generated data and real-world measured data in Sec. 5, and some practical considerations are discussed. Finally, conclusions and future work are outlined in Sec. 6.

2. Problem Formulation and Pre-filtering

A. Signal Model and Problem Statement

Consider a complex baseband signal as follows

$$x(t)=\sum_k a_k\, p(t-kT) \qquad (1)$$

where a_k is the transmitted symbol with $|a_k|=1$, and $p(t)$ is the baseband shaping. If $x(t)$ is passing through a flat fading channel, the complex envelope of the received signal can then be written as

$$y(t)=f(t)\,x(t)+w(t) \qquad (2)$$

where $f(t)$ is the multiplicative fading process of interest, and $w(t)$ is additive white Gaussian noise. According to [3], we assume that the complex fading process consists of multiple complex sinusoids as

$$f(t)=\sum_{i=1}^{P} A_i e^{j(2\pi f_i t+\theta_i)} \qquad (3)$$

which can be interpreted as P incident rays with different path attenuations A_i, Doppler shifts f_i, and phase shifts θ_i, $i=1,\dots,P$. In addition, the maximum possible Doppler shift (fading rate) can be assumed known, since it is given by

$$f_{D\max}=f_c\cdot v_{\max}/c \text{ (Hz)} \qquad (4)$$

where f_c is the carrier frequency, v_{max} is the maximum vehicle speed, and c is the speed of light. For example, with f_c = 1.9 GHz and v_{max} = 60 mph, f_{Dmax} is set to about 180 Hz.

We assume that the symbol rate is much greater than the fading rate and the overall pulse shaping satisfies the zero ISI condition at the symbol sampling points. So after receive filtering and sampling at the symbol rate $R=1/T$, the resulting discrete-time samples are given by

$$y(t)\,|_{t=kT}=y(k)=f(k)\,a_k+w(k) \qquad (5)$$

Let us assume that the receiver has correctly detected the symbol a_k. Then by multiplying the received sample $y(k)$ by $\hat{a}_k^{\,*}=a_k^*$, the modulation can be removed, yielding

$$z(k)=y(k)\,\hat{a}_k^{\,*}=f(k)+w'(k) \quad k=0,1,2,\dots,N \qquad (6)$$

where $w'(k)$ is still an additive white Gaussian noise with the same variance as $w(k)$. We also normalize the sinusoidal frequencies for the discrete-time data with respect to the symbol rate (sampling frequency), and denote these frequencies by $\bar{f}_i=f_i/R=f_i\,T$.

With such a record of noisy fading data, we now state the problem as follows :

Given the data record $\{z(k), k=0,\dots,N\}$ and maximum Doppler frequency f_{Dmax},

(1) Estimate the parameters $\{A_i, \bar{f}_i, \theta_i\}$ in the flat fading model.

(2) Predict the fading process $f(k)$ for $k=N+1,\dots,N+L$.

B. Low-Pass Pre-filtering

Since the normalized maximum frequency component of the discrete-time samples $f(k)$ is no more than $f_{Dmax}T$, we can first filter the out-of-band noise in $z(k)$. This leads to an increase in *fading power-to-noise power ratio* (FNR). In fact, the conventional averaging method can be regarded as a low-pass filter (LPF) with an impulse response of a rectangular window. In [7] it has been shown that the selection of window length and window type has a influence on the overall performance. So here we will use the formal filter design technique for low pass pre-filtering. First, the LPF should be linear phase since it is to pass $f(k)$ without distortion. Thus we use an FIR filter rather than an IIR filter. Then, among the various FIR filters, we consider the Parks-McClellan optimal equiripple filter which satisfies the following specifications :

Passband edge : $f_{Dmax}T$

Stopband edge : $f_{Dmax}T+f_{TB}$ (f_{TB} is the width of transition band)

Maximum passband ripple : ρ dB (should be small)

Minimum stopband attenuation : D dB

The transfer function of this filter can be written as

$$B(z)=\sum_{i=0}^{L} b(i)z^{-i}\,. \qquad (7)$$

Suppose that an ideal LPF is used. Then the improvement factor in FNR by using the ideal LPF is $10\log(R/f_{Dmax})$. For a practical FIR LPF, the improvement factor is about 10 dB, mainly depending on the filter order L.

When applying the LPF to the data record, two things should be noted. First, the output noise component $v(k)$ becomes a colored moving average (MA) noise with known autocorrelation function :

$$R_v(l)=\sigma_w^2\sum_{i=0}^{L-l} b(i)b(i+l) \text{ for } |l|<L \text{ and zero elsewhere} \quad (8)$$

that is assumed known. Second, there exists a group delay (misalignment) of $L/2$ between the true fading process $f(k)$ and the LPF output $g(k)$. Without time shifting, this can lead to a combiner weight bias and can therefore degrade the BER performance [7], especially in fast fading situations.

We present the sinusoidal modeling and prediction method below.

3. Sinusoidal Modeling for Fading Processes

The present problem is similar to the harmonic retrieval problem that has been extensively discussed over the past 20 years. Thus there are numerous existing methods [8] to solve this problem. Basically, they can be classified into two categories : FFT-based methods and model-based high-resolution methods. Since the fading rate is much less than the symbol rate, the frequencies of the component sinusoids are closely spaced around zero. Thus we resort to the high resolution methods. In the literature, we note that the ROOT-MUSIC method [6] matches our needs and is computationally moderate. For this reason, we consider it in this paper. Below we give a brief introduction of it.

As a variant of the well-known MUSIC method, ROOT-MUSIC does not do spectral peak finding. First, a K-by-K sample correlation matrix is calculated as

$$\mathbf{R}=\mathbf{G}^*\mathbf{G}^H \quad (9)$$

where $\mathbf{G}$ is the forward-backward data matrix constructed from the LPF output data $g(k)$. Assuming that the number of sinusoids is P ($P < K-1$), then the noise subspace is obtained as

$$span\{\mathbf{V}_n\}, \quad \mathbf{V}_n=[\mathbf{v}_{P+1}\ \mathbf{v}_{P+2}\ \ldots\ \mathbf{v}_K] \quad (10)$$

where $\mathbf{V}_n$ consists of the K-P smallest eigenvectors of $\mathbf{R}$. Let $\mathbf{Q}=\mathbf{V}_n\mathbf{V}_n^H$ and

$$c_i=\sum_{k=1}^{K-i}\mathbf{Q}_{k,k+i} \text{ and } c_{-i}=\sum_{k=1}^{K-i}\mathbf{Q}_{k+i,k} \quad for\ i=0,1,2,\ldots,K-1 \quad (11)$$

We note that $c_i^* = c_{-i}$, and form the polynomial equation

$$c_{-K+1}+c_{-K+2}z^{-1}+\cdots+c_0z^{-K+1}+c_1z^{-K}+\cdots+c_{K-1}z^{-2(K-1)}=0 \quad (12)$$

Solving this equation gives $2(K-1)$ roots having reciprocal symmetry with respect to the unit circle. Denote the P roots that are outside and also nearest to the unit circle as z_1, ..., z_P. Then the frequency estimates for $\tilde{f}_i$ (normalized with respect to R) are given by

$$\hat{f}_i = \arg(z_i)/2\pi \qquad i=1,2,\ldots,P \quad (13)$$

where $\arg(z_i)$ denotes the principal argument (in radians) of z_i. It should be pointed out here that the method needs to know the number of sinusoids *a priori*. However, in next Section we will use a root location constraint to avoid this problem.

Once the frequency estimates have been obtained, the complex amplitudes $E_i = A_i e^{j\theta_i}$ can be found by a linear least-square (LS) fit of the following matrix-vector equation

$$\mathbf{A}\,\mathbf{E} = [\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_P]\,\mathbf{E} = \mathbf{g} \quad (14)$$

where $\mathbf{a}_i = [1\, e^{j2\pi\hat{f}_i} \ldots e^{j2\pi\hat{f}_i N}]^T$ for $i = 1,\ldots,P$, $\mathbf{E} = [E_1 \ldots E_P]^T$ is the complex amplitude vector to be found, and $\mathbf{g} = [g(0)\ g(1) \ldots g(N)]^T$. The LS solution of (14) is given by $\hat{\mathbf{E}} = \mathbf{A}^{\#}\mathbf{g}$, where $\mathbf{A}^{\#}=(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$ is the pseudo inverse of $\mathbf{A}$. In this way, the parametric sinusoidal model for the fading process is obtained.

4. Fading Prediction

Using the fading model determined by the above method, fading prediction can be done. The overall concept for fading modeling/prediction is illustrated in Fig1. However, the parameter estimation or model fitting suffers from error due to the presence of noise, and nonstationarity due to the time-varying sinusoidal parameters with real-world fading. Below we consider this issue.

A. Performance measures for fading prediction

Let the predicted (extrapolated) fading process be denoted by $\hat{f}(k)$ for $k > N$. When the true fading $f(k)$ is known, as in the computer simulation case, then the fading prediction error is

$$e(k)=f(k)-\hat{f}(k) \qquad k>N \quad (15)$$

On the other hand, for the real-world measured data case, the true fading is not known exactly. Thus we use the LP filtered fading process $g(k)$ instead of $f(k)$, and define the prediction error as

$$e_r(k)=g(k)-\hat{f}(k) \qquad k>N \quad (16)$$

Now, for the computer simulation case, assume there are M independent (Monte-Carlo) experiments under the same fading conditions, and denote the error for the i-th experiment as $e_i(k)$. We adopt the normalized mean square error (NMSE) as a performance measure, i.e.

$$NMSE_f(k)=\frac{\frac{1}{M}\sum_{i=1}^{M}|e_i(k)|^2}{\frac{1}{MN}\sum_i\sum_n|f(n)|^2} \qquad k>N \quad (17)$$

where the denominator represents the average power of the simulated fading process.

Since the complex fading process can be divided into an envelope fading and a phase fading process, we can similarly define two performance measures for the fading envelope

$$NRMSE_E(k)=\frac{\sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(|f_i(k)|-|\hat{f}_i(k)|\right)^2}}{\sqrt{\frac{1}{MN}\sum_i\sum_n|f(n)|^2}} \qquad (18)$$

and for the fading phase

$$RMSE_P(k)=\sqrt{\frac{1}{M}\sum_{i=1}^{M}m_p\left(\angle f_i(k)-\angle\hat{f}_i(k)\right)^2} \qquad (19)$$

where $\angle f_i(k)$ denotes the phase of $f_i(k)$, and is in the range of -180^o to 180^o. In (19), $m_p(\theta)$ is a phase correction function used for subtracting or adding 360^o when the phase error is larger than 180^0 or less than 180^0, respectively. That is,

$$m_p(\theta)=\begin{cases}\theta+360^o & if \quad \theta<-180^o\\ \theta-360^o & if \quad \theta>+180^o\\ \theta & otherwise\end{cases} \qquad (20)$$

B. Sensitivity of prediction performance to estimation error

Consider the case of a single complex sinusoid as follows

$$f(k)=A_ie^{j(2\pi\bar{f}_ik+\theta_i)}=A_ie^{j\theta_i}e^{j2\pi\bar{f}_ik}=E_ie^{j2\pi\bar{f}_ik} \qquad (21)$$

Note that $f(k)$ is linear in the complex amplitude E_i, but is nonlinear in the frequency $\bar{f}_i$. We have

$$\left|\frac{\partial f(k)}{\partial\bar{f}_i}\right|=2\pi k|f(k)| \quad \text{or} \quad \left|\frac{\partial f(k)}{f(k)}\right|=2\pi k|\partial\bar{f}_i| \qquad (22)$$

Thus a small error in the frequency estimate can result in an unbounded change in $f(k)$ with time. This is also true for the multiple sinusoids case. Thus, fading prediction performance is quite sensitive to frequency error, becoming less accurate for longer prediction.

C. Performance improvements

Because of the above problem, we consider the following modifications of the ROOT-MUSIC method to enhance its performance for fading prediction.

1. Roots location constraint

Since the fading process can be described as a very narrowband LP process consisting of multiple sinusoids, the frequencies of these sinusoids will be closely clustered in the neighborhood of 0 Hz. Furthermore, the maximal possible sinusoidal frequency can be set to the maximum Doppler frequency, f_{Dmax}. The above *a priori* information corresponds to a constraint on the permissible location of the candidate roots obtained by ROOT-MUSIC. More specifically, denote the roots that are outside the unit circle as z_1, ..., z_{K-1}. Then we use a root discrimination rule to select the admissible roots as follows:

$$z_i \textit{ is admissible if } \left\{\frac{|\arg(z_i)|}{2\pi}<f_{D\max}T=\varepsilon\right\} \text{ AND } \left\{\,|\,z_i\,|-1<\delta\right\} \text{ for } i=1,\dots,K-1 \qquad (23)$$

where the second condition is used to exclude some spurious roots which are too far from the unit circle. In this way, we also obtain the number of modeling sinusoids from the data.

2. Control-point constraint

This constraint ensures that the estimated fading process coincides with the LP filtered fading data at some selected control points. Such points may be boundary points, level crossing points, peak points, etc, depending on the purpose and situation. Since the frequency estimation method is highly nonlinear, this constraint is difficult to embed in the frequency estimation method. However, it can be incorporated into the linear LS fitting eq.(14) as a linear constraint. Let's formulate this constrained LS problem as follows :

$$\min_{\mathbf{E}}\|\mathbf{AE}-\mathbf{g}\|^2 \qquad (24)$$
$$\text{subject to } \mathbf{CE}=\mathbf{d}$$

where $\mathbf{A}$ is a known matrix constructed from the frequency estimates, and the number of rows of $\mathbf{C}$ denotes the number of control points. Assume there are L control points, and denote the indices of control points as $c_1,\dots c_L$. Then the i-th row of $\mathbf{C}$ is equal to the c_i-th row of $\mathbf{A}$, and the same rule holds between the two column vectors $\mathbf{d}$ and $\mathbf{g}$.

To solve the above problem, we use the method of Lagrange multipliers, which yields the solution as follows :

$$\mathbf{E}=(\mathbf{A}^H\mathbf{A})^{-1}(\mathbf{A}^H\mathbf{g}-0.5\,\mathbf{C}^H\boldsymbol{\lambda}) \qquad (25)$$

where the Lagrange multiplier vector $\boldsymbol{\lambda}=[\lambda_1 \dots \lambda_L]^T$ is given by

$$\boldsymbol{\lambda}=[\mathbf{C}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{C}^H]^{-1}\,[2\mathbf{C}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{g}-2\mathbf{d}]\,. \qquad (26)$$

5. Experiments and Performance Evaluation

A. Results based on computer generated data

First, we test the short-term prediction performance of the proposed method by using the Jakes simulated Rayleigh fading process. Some important parameter settings in this scenario are as follows :

- fading rate : f_{Dmax} =180 and 250 Hz
- No. of sinusoids in Jakes model : $P = 8$
- Fading process power : $P_f = 1$
- Modulation format : QPSK with $a_k \in \left\{\pm\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}\right\}$
- Sampling rate : 48.6k samples/sec
- SNR : 10, 15, 20, 25 dB
- LPF: 28th order equiripple FIR filter
- No. of modeling points by ROOT-MUSIC : $N+1 = 20$
- Dimension of correlation matrix $\mathbf{R}$: $K = 10$
- Roots location constraint for ROOT-MUSIC : $\varepsilon = 1.2 f_{Dmax}T$ and δ =0.1
- Control point constraint for LS fitting : ending point of the modeling interval
- Monte-Carlo runs : $M = 500$ for each SNR

The simulation results are shown in Fig. 2 and 3, which show the $NMSE_f$, $NRMSE_E$, and $RMSE_P$. The fluctuation in the $RMSE_P$ curves is due to some Monte-Carlo trials that give large phase errors when the fading envelope is very small. Except for this situation, it is seen that with only 20 modeling points, short-term prediction of the next 10 points is very good, i.e., with an $NRMSE_E$ under 0.1 and $RMSE_P$ less than 10 degrees.

B. Results based on real-world data

Here, we used two real-world IS-136 data sets collected during a Lucent/AT&T field trial [9]. For the data collection, after down conversion, the received I/Q baseband data were first filtered by a squared root raised-cosine filter with a roll-off factor of 0.35, and then sampled at 48.6 kHz. With the knowledge of the correct symbol sequence, we removed the modulation from the data. Each of the data sets then consisted of 45 time slots, which amounts to a total of 45×168= 7560 sample points (0.156 seconds).

We converted the measured fading data into MATLAB data format and applied the same LPF to the data sets as in the simulated case. We then applied the aforementioned fading modeling and prediction method to the preprocessed real-world fading data. The results are shown in Fig.4. As can be seen, the short-term prediction error is lower for the real-data case than for the simulated case. Although the fading rate is slightly lower than that in the simulated case (≈ 150 Hz), this is mainly due to fewer strong sinusoids than in the simulated case (with 8 sinusoids). However, we also observed that long-term prediction for the real-world data can be significantly worse than that with computer simulated fading data, This is due to the non-stationarity of the real-world data.

6. Concluding Remarks

We have studied the problem of sinusoidal modeling and prediction of fast fading processes. Some promising results have been presented, showing good short-term prediction. However, this study is by no means exhaustive, but is a preliminary effort. Many questions need to be answered in the future. We will continue to work on more real-world data, attempt to find other methods with better performance, and examine the potential for applying this technique in practical applications.

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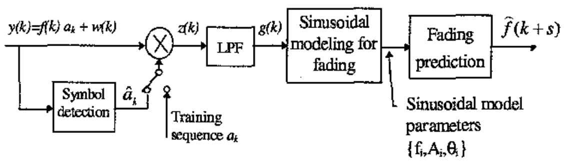
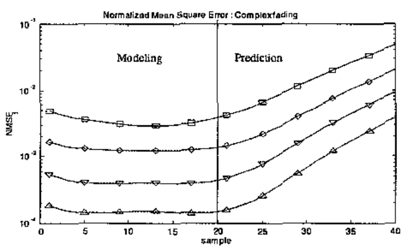


Fig.1 Block diagram of the fading modeling/prediction method



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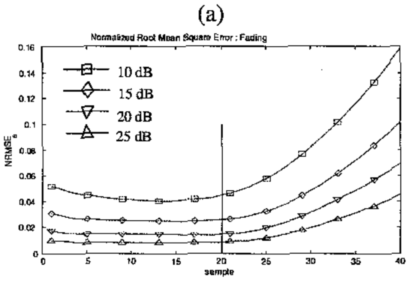

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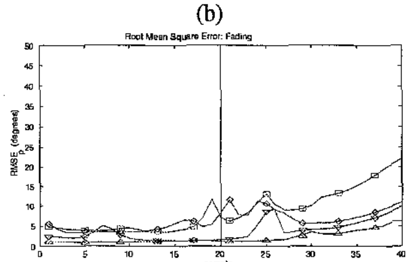
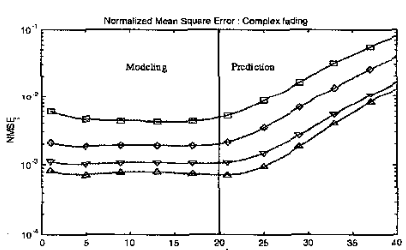


(c)

Fig.2 The simulation results for f_{Dmax} = 180 Hz.
(a) $NMSE_f$ (b) $NRMSE_E$ (c) $RMSE_P$



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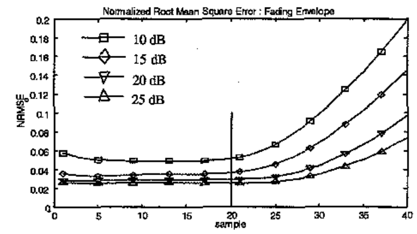

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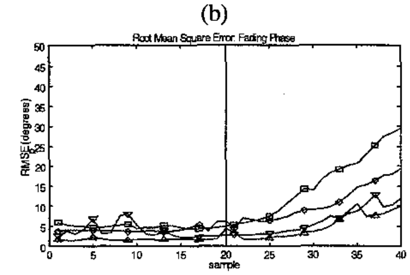
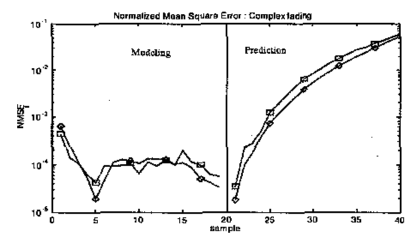


(c)

Fig. 3 The simulation results for f_{Dmax} = 250Hz.
(a) $NMSE_f$ (b) $NRMSE_E$ (c) $RMSE_P$



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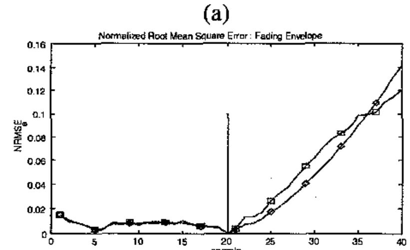

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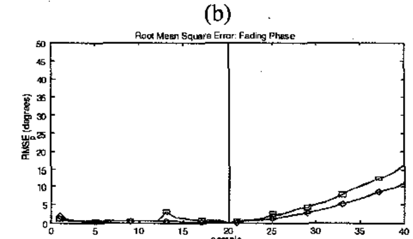

(c)

Fig. 4 Performance results for two real-world measured data sets (a) $NMSE_f$ (The value at the *20th* sample point is $-\infty$, due to the ending point constraint.) (b) $NRMSE_E$ (c) $RMSE_p$