Adaptive Antenna Arrays Using Sub-space Techniques in a Mobile Radio Environment with Flat Fading and CCI.

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Abstract:
In this paper we compare the performance of the minimum mean square error (MMSE) and maximum signal to interference plus noise ratio (MSINR) cost functions in a flat fading channel with co-channel interference (CCI). We investigate the reduced rank and full rank weighted sub-space techniques. We observe that using the reduced rank approximation technique on MMSE is equivalent to MRC in a flat fading channel without CCI. For the CCI case we found the full rank weighted sub-space technique applied to MSINR performs best. Through reduced rank approximations of the covariance matrices, the signal sub-space performs well for the MMSE and the noise sub-space performs well for the MSINR. However, the weighted sub-space performance was close to the known rank case and much better than the conventional techniques.

1. Introduction:

The need for improved communications and increased system capacity is never ending. Wireless system designers have several options available from choosing a modulation scheme, adding Forward Error Correction, including spatial antenna diversity, etc. We will concentrate on increasing the number of antennas for reception. It is well known that antenna diversity increases the output SNR, lowering the bit error rate (BER) and thus producing better performance. Three techniques available are: Switching/Selection (SS), Equal Gain (EG) Combining and Maximal Ratio Combining (MRC).[4] However when CCI is present in the channel the above mentioned techniques are no longer optimum in minimizing the BER. The optimum combiner is the Adaptive Antenna Array (AAA).[10]

As a demonstration vehicle, we chose the IS-136 TDMA NADC standard. This cellular system uses π/4-DQPSK at a bit rate of 48.6 kbps.[5] Our choice of detection technique is the differential detector.[13] The BER was measured over 10 million bits. The AAA receiver has M antennas. The received signal vector is represented as

$$\underline{x} = \left[x_1, x_2, \cdots, x_M\right]^T$$

where T denotes transpose. The weight vector is defined as $\underline{w}$, so the AAA output signal is

$$y = \underline{w}^* \underline{x} \quad (1)$$

where * denotes complex conjugate transpose.

Our simulation results show in a flat fading channel without CCI, the MMSE and MSINR based weights have poorer performance than MRC. Note the degradation with respect to the ideal weights is due to the sample support problem, which occurs when the number of weights to be determined (M) approaches the window size (N). Moreover, when CCI is present, we show the BER performance of MSINR and MMSE are approximately equivalent, but much better than MRC. Further improved performance can be achieved if the number of CCI's is known. In practice this is not known so we use the weighted sub-space technique and achieve performance very close to the known rank case. Lastly, reduced rank approximation of the covariance matrices is investigated for both MSINR and MMSE, and compared to the full rank weighted sub-space approach.

Section 2 describes the two cost functions and the sub-space techniques. Section 3 presents BER results for the frequency flat fading channel. In section 4 we present BER results in the flat fading with CCI channel. Section 5 presents the weighted sub-space results, and conclusions are in section 6.

2. Cost Function Description:

In this section we compare the two cost functions used to derive the array weights.

2.1 MMSE Cost Function:

The MMSE cost function minimizes the MSE between the array output and desired signal, d. This leads to the following equation for the antenna weights [9]

$$\underline{w}_{MMSE} = \hat{R}_{xx}^{-1} \hat{\underline{r}}_{xd} \quad (2)$$

where $\hat{R}_{xx}$ is the received signal covariance matrix and $\hat{\underline{r}}_{xd}$ is the cross correlation vector. We use the sample mean estimator which gives us the following estimate

$$\hat{R}_{xx} = \frac{1}{N}\sum_{i=1}^{N} \underline{x}(t+t_i)\underline{x}^*(t+t_i) \quad (3)$$

and the following estimate of the cross correlation vector

$$\hat{\underline{r}}_{xd} = \frac{1}{N}\sum_{i=1}^{N} \underline{x}(t+t_i)d^*(t+t_i) \quad (4)$$

We have used N to denote the window size, i.e., the number of time samples (or symbols) used in the calculation.

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2.2 MSINR Cost Function:

The MSINR cost function maximizes the array output SINR. This leads to the following equation for the antenna weights

$$\underline{w}_{MSINR} = \hat{R}_{I+N}^{-1}\,\hat{\underline{r}}_{xd} \quad (5)$$

where the received signal interference plus noise covariance matrix is given as (with the variable t omitted)

$$\hat{R}_{I+N} = \frac{1}{N}\sum_{i=1}^{N}\left(\underline{x}-\hat{\underline{h}}d\right)\left(\underline{x}-\hat{\underline{h}}d\right)^{*} \quad (6)$$

where $\hat{\underline{h}}$ is an estimate of the desired signal's channel response, which we set equal to the cross correlation vector. With ideal estimation, the array weights for MMSE and MSINR are equivalent (within a scalar factor) and produce the same BER performance.

2.3 Reduced Rank and Sub-space Techniques:

In this section we make use of some special properties of the covariance matrices described above. In particular they are Hermitian, as such Normal matrices and unitarily diagonalizable. Using the Eigen Spectral Decomposition (ESD) Theorem we can rewrite the covariance matrix as [2]

$$\hat{R} = \sum_{i=1}^{M}\lambda_i\,\underline{v}_i\,\underline{v}_i^{*} \quad (7)$$

where λ_i's are the associated eigenvalues of the matrix, $\hat{R}$, and $\underline{v}_i$'s are the associated eigenvectors. Note $R=R_{I+N}$ for MSINR and $R=R_{xx}$ for MMSE. We know the vector space of R consists of M linearly independent vectors. We can go one step further to classify them into sub-spaces: the signal sub-space and the noise sub-space. Let's define N_s to be the dimension of the signal sub-space. By signal sub-space we mean the desired + interference for MMSE and interference for MSINR. The MMSE weights can thus be rewritten as

$$\underline{w}_{MMSE} = \left(\sum_{i=1}^{N_s}\frac{1}{\lambda_i}\underline{v}_i\underline{v}_i^{*} + \sum_{i=N_s+1}^{M}\frac{1}{\lambda_i}\underline{v}_i\underline{v}_i^{*}\right)\hat{\underline{r}}_{xd} \quad (8)$$

Note the eigenvectors for R^{-1} are the same as those of R. With ideal estimation, the second summation should identically equal zero since $\hat{\underline{r}}_{xd}$ is in the signal sub-space. This leads to the reduced rank approximation to the covariance matrix. In particular to the MMSE cost function, we can define the reduced rank sub-space weights by using only the eigenvectors associated with the largest N_s eigenvalues. The weights are given as

$$\underline{w}_{MMSE-SS} = \left(\sum_{i=1}^{N_s}\frac{1}{\lambda_i}\underline{v}_i\underline{v}_i^{*}\right)\hat{\underline{r}}_{xd} \quad (9)$$

where N_s is equal to P+1 for the MMSE cost function and P denotes the number of CCI.

In consideration to the MSINR cost function, we use the noise sub-space since this is orthogonal to the interference sub-space and can cancel out the respective interference eigenvalues. This technique is the Eigencanceller, [3] whose weights equal

$$\underline{w}_{MSINR-EC} = \underline{w}_{MSINR-NS} = \left(\sum_{i=N_s+1}^{M}\frac{1}{\sigma_n^2}\underline{v}_i\underline{v}_i^{*}\right)\hat{\underline{r}}_{xd} \quad (10)$$

where N_s is equal to P for the MSINR cost function and σ_n^2 is the noise variance. We can do better in performance through the use of full rank, signal sub-space techniques. In particular let's discuss the MSINR with averaged noise sub-space, whose weights are given as

$$\underline{w}_{MSINRw/AVG-NS} = \left(\sum_{i=1}^{N_s}\frac{1}{\lambda_i}\underline{v}_i\underline{v}_i^{*} + \frac{1}{\sigma_n^2}\sum_{i=N_s+1}^{M}\underline{v}_i\underline{v}_i^{*}\right)\hat{\underline{r}}_{xd} \quad (11)$$

Here we assumed the number of CCI present are known. A work around is to use the Weighted Sub-space technique shown below.

3. Flat Fading without CCI BER Results:

In this section we will represent the received signal as

$$\underline{x} = \underline{h}s + \underline{n} \quad (12)$$

where $\underline{h}$ is an Mx1 channel vector, s is the desired signal and $\underline{n}$ is an Mx1 noise vector. Each element of the channel vector is a complex Gaussian random variable, whose magnitude is Rayleigh distributed and phase is uniformly distributed from 0 to 2π. The noise is AWGN. We observe the performance for 3 Doppler spreads, f_d= 30Hz, 80Hz and 190Hz. Moreover, we assume the antennas are spaced far enough apart to obtain independent fading on each branch.

For this channel, we observed the performance of MSINR and MMSE to be worse than that of MRC. However, if we knew *a priori* there was no interference, we can use the reduced rank approximation techniques on MMSE and MSINR to obtain equal performance to MRC. Using the reduced rank approximation technique on the MMSE (N_s=1) is equivalent to MRC. For the MSINR cost function, theoretically R_{I+N} is equivalent to R_{nn}, so we have the weight equation which is equivalent to the MRC method. We can also use unequal windowing in estimating R_{nn}. The cross correlation vector can't have a large window since the channel changes during the estimation window. However R_{nn} should be a diagonal matrix since it only contains noise. Here a larger averaging window is preferred. We can force all the eigenvalues to equal the noise variance or its estimate. In fact any constant, say 1, will diagonalize the matrix to give

$$\underline{w}_{MRC} = \hat{\underline{r}}_{xd} \quad (13)$$

3.1 M=2 & M=5 Performance Discussion:

Here we discuss the BER performance of the AAA with M = 2 & 5, using the MMSE (2) and MMSE-SS (9) with N_s=1. For all f_d, we found the MMSE weights to perform worse than the MRC for this flat fading channel. However, the performance of AAA using MMSE-SS is equivalent to the

MRC method. So the reduced rank approximation improved the performance of the MMSE based solution to that of the MRC. We observed that the f_d=190Hz channel favors N=7, f_d=80 Hz favors N=15 and f_d=30 Hz favors N=30 symbols. Hence it would make sense for the receiver to have a Doppler estimator to adapt the window size to the channel condition. At N=4 symbols the performance is f_d independent since the major source of distortion is due to estimation error. For N > 30, there is f_d dependency which is due to the channel changing during the estimation window. Lastly, we see using the MMSE-SS technique, the performance "flattens" so we are somewhat less sensitive to the value of N chosen. For lower Doppler spreads (slowly varying channel) the array weights can be more accurate since we can use larger estimation window sizes. The challenge occurs for fast fading channels which forces us to use the small window sizes. For M=5, we saw similar behavior as for the M=2 case, for both the full and the reduced rank signal sub-space methods.

In summary, for the flat fading channel we observed the performance of MSINR and MMSE to be worse than that of MRC. However, applying the reduced rank approach (MMSE-SS) or unequal windowing to MSINR, performance is similar to MRC.

4. Flat Fading with CCI BER Results:

In this section we will represent the received signal as

$$\underline{x} = \underline{h}s + \sum_{i=1}^{P} \underline{h}_{I_i} s_{I_i} + \underline{n} \tag{14}$$

where $\underline{h}$ is an Mx1 desired signal channel vector, $\underline{h}_{I_i}$ is the i[th] Mx1 interfering signal channel vector, s is the desired signal, s_{I_i} is the i[th] interfering signal and $\underline{n}$ is an Mx1 noise vector. Here we assumed P equal power CCI's in the channel. The channel vector is a complex Gaussian random variable. The noise is AWGN. We define signal to interference plus noise (SINR) as

$$SINR = \frac{SNR}{1 + \sum_{i=1}^{P} INR} \tag{15}$$

As the interference to noise ratio (INR) increases the AAA performs better than MRC.[6] [10] Below we have set the INR to equal 10dB.

4.1 M=2 Performance Comparison:

In figure 1 we plot the performance of MSINR and MMSE with 1 CCI (P=1) for f_d=190Hz, using N=15 and N=7 symbols. We notice for N=7 symbols, the AAA can perform much better than the MRC. Here we see MSINR & MMSE perform approximately 3dB better than MRC for BER=1E-2.

4.2 M=5 Performance Comparison:

For P=1, using the MMSE-SS produced an SINR improvement of approximately 0.3dB at a BER=1E-3. Hence the reduced rank approximation for R_{xx} can lead to improvement when P is low compared to M. We investigated the performance of the MMSE, MSINR, MSINR-EC and MSINR w/AVG-NS for P=1. MSINR w/AVG-NS performed the best, followed by MSINR-EC and MSINR.

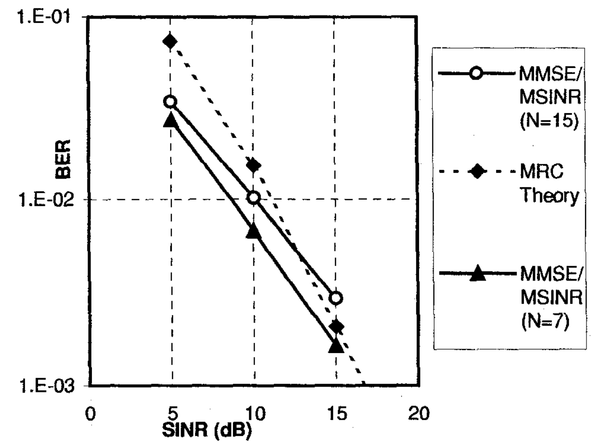

Figure 1: MMSE & MSINR Performance with 1 CCI.

We increased P to 3 and the MSINR-EC performed worse than MSINR. The performance of MSINR-EC is extremely dependent on P. Hence this reduced rank approximation to R_{I+N} can improve performance when the number of CCI is small, compared to M. In order to observe the performance dependency on N we plot figure 2 for the 1 CCI case, for SINR=2.5dB. The performance improvement over MMSE is more noticeable at low values of N.

As P increased the performance of the MSINR-EC degraded to be worse than MMSE. We noticed the MSINR-EC works well when the dimension of the noise sub-space is greater than P. Unequal windowing didn't affect the performance of MSINR w/AVG-NS. However for MSINR, performance improves, but is still worse than the sub-space techniques. We have found 7 symbols for $\hat{r}_{xd}$ and 15 symbols for $\hat{R}_{I+N}$ gave improved performance, compared to equal windowing.

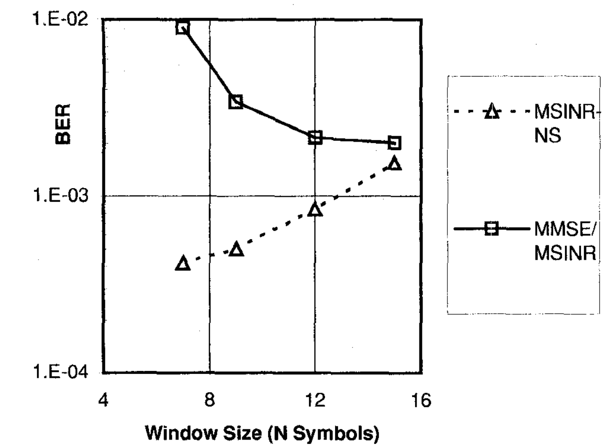

Figure 2: MMSE & MSINR BER Results for 1 CCI.

As P increases, the performance of MSINR-NS degrades more rapidly. Also depending on the number of CCI present, different values of N improve performance. For MMSE, slight improvement occurs using the reduced rank technique with the correct rank estimate. Significant degradation occurs when this rank estimate is below P+1. For MSINR-EC, we notice a correct rank estimate improves performance and an incorrect estimate causes significant degradation in BER.

Figure 3 shows the BER versus the signal sub-space dimension (N_s) for values of P=1 to 4 in (11). Here we fixed SINR=5dB and N=7 symbols. As expected, the minimum BER is achieved when the correct dimension of the sub-space is chosen, N_s=P. (This figure assumes $\hat{h} = r_{xd}$, similar results were obtained when we set $\hat{h} = \hat{r}_{xd}$.)

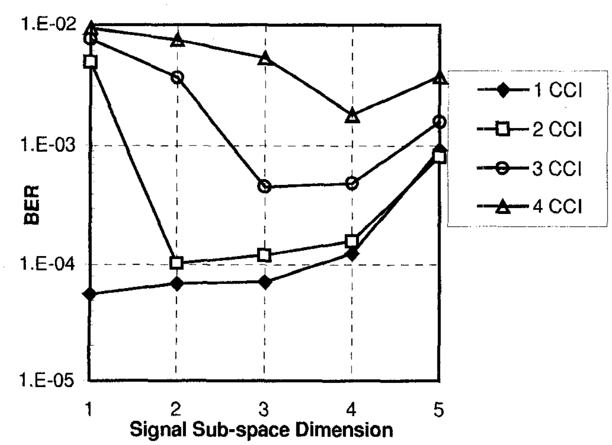

Figure 3: MSINR Full Rank Sub-space Investigation.

BER performance improvement can be obtained through full rank, sub-space processing. Note that this performance improvement is achieved by forcing a particular structure to the covariance matrices. For example, if we correctly choose the signal sub-space dimension, we have forced the matrix to have the proper structure. By this we mean the interference matrix will have rank P and the noise sub-space eigenvalues are all set equal to noise variance.

5. Weighted Sub-space (WSS) Performance:

Above we have assumed *a priori* knowledge of the rank of the interference covariance matrix. In practice this must be estimated. We can get around this problem by the weighted sub-space (WSS) technique. [11] [12] In the WSS technique, the sub-spaces are weighted according to the eigenvalues. Let's further define the eigenvalues as

$$\lambda_i = \tilde{\lambda}_i + \sigma_n^2 \qquad (\forall i \in 1,\dots,M) \qquad (16)$$

where $\tilde{\lambda}_i$ is an eigenvalue corresponding to the signal sub-space, in the absence of noise. The WSS technique performs the following function: If $\tilde{\lambda}_i$ is large, we leave it alone (since it corresponds to large interference), or if $\tilde{\lambda}_i$ is small we force it to approach zero (since it corresponds to a very weak interferer or it is non-existent). We can write the interference + noise matrix as

$$\hat{R}_{I+N} = \sum_{i=1}^{N_s} f\left(\tilde{\lambda}_i\right)\tilde{\lambda}_i \underline{v}_i \underline{v}_i^* + \hat{\sigma}_n^2 \mathrm{I} \qquad (17)$$

where $f(\tilde{\lambda}_i)$ is a non-linear function that behaves as discussed above. We used the ATAN(x) function, shifted and appropriately scaled, which we found to be robust over a wide range of nonlinear functions. Our simulations have shown this technique works very well (minimal BER degradation) compared to the known rank case, as shown in figure 4. Here we see the tremendous benefit of the WSS technique to overcome the sample support problem.

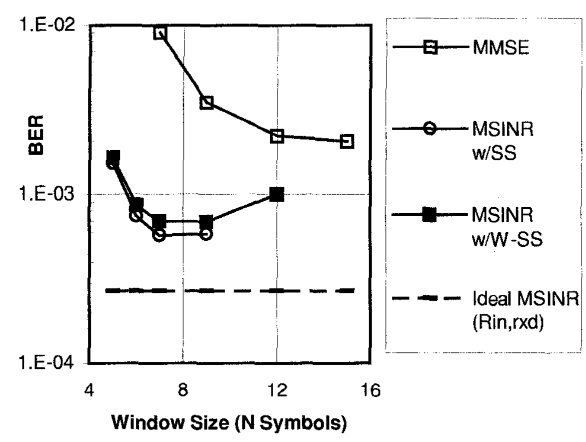

Figure 4: WSS Results for 1 CCI, SINR=2.5dB.

The weighted sub-space technique performance with different number of CCI is shown in figure 5. We see how well the WSS technique performs compared to the known rank performance. We noticed as P increases the eigencanceller degrades more rapidly than the other techniques and the sub-space improvement over the conventional techniques decreases.

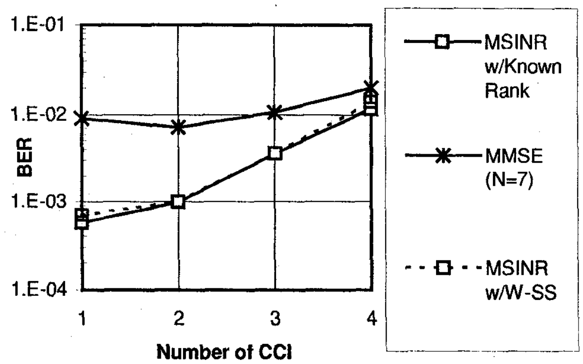

Figure 5: WSS Performance, SINR=2.5dB, N=7, f_d=190Hz.

Figure 6 compares the AAA performance for M=2 and M=5 with 1 CCI. We see the optimum AAA has approximately an 8dB improvement over M=5 MRC for BER=1E-3. Using the WSS method we are approximately 1.5dB worse than this optimum known rank case and 2 dB better than the conventional MSINR weights. Hence this WSS technique offers tremendous performance improvements.

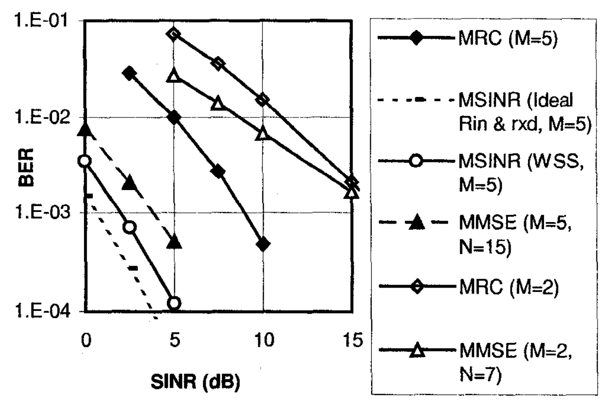

Figure 6: M=2 & 5 AAA BER Performance Comparison with 1 CCI.

6. Conclusions:

For flat fading without CCI, we have shown the conventional MMSE and MSINR methods are worse than MRC. Use of the reduced rank improves performance to equal MRC and therefore these techniques don't produce any degradation for this channel condition.

For the CCI case, we showed that MSINR & MMSE are better than MRC, as expected. We can obtain better performance than the eigencanceller using MSINR w/AVG-NS. Use of the signal sub-space for MMSE and the noise sub-space for MSINR has some limitations as P increases. Specifically, as P approaches M, the reduced rank techniques degrade faster than the sub-space techniques. Moreover, for this large P case the unequal windowing technique on MSINR produces better results than the reduced rank. The channel estimation technique seems to be the dominant cause of BER degradation. Best performance can be obtained if the number of CCI is known, but in practice this is unknown. For this case, the weighted sub-space (WSS) method performs close to this known rank case and much better than the conventional methods.

References:

[1] I. S. Reed, J. D. Mallett and L.E. Brennan, "Rapid Convergence Rate in Adaptive Arrays," IEEE Trans. Aerospace and Elec. Sys., Vol. 10, pp. 853-863, Nov. 1974.

[2] P. Lancaster and M. Tismenetsky, *The Theory of Matrices*, Academic Press, 1985.

[3] X. Wu and A. M. Haimovich, "Adaptive Arrays for increased Performance in Mobile Communications," IEEE PIMRC 1995, Toronto Canada, pp. 653 - 657.

[4] W. C. Jakes, *Microwave Mobile Communications*, New York, John Wiley, 1974.

[5] Cellular System Dual-Mode Mobile Station-Base Station Compatibility Standard, EIA/TIA, TR45, IS-54, Jan. 9, 1992.

[6] J. H. Winters, "Signal Acquisition and Tracking with Adaptive Arrays in Digital Mobile Radio System IS-54 with Flat Fading," IEEE Trans. Veh. Tech. Vol VT-42, pp. 377-384, November 1993.

[7] T. Boros, G. G. Raleigh and M. A. Pollack, "Adaptive Space-Time Equalization for Rapidly Fading Communication Channels," IEEE 1996, pp. 984 - 989.

[8] G. G. Raleigh and A. Paulraj, "Time Varying Vector Channel Estimation for Adaptive Spatial Equalization," IEEE 1995, pp. 218 224.

[9] R. Monzingo and T. Miller, *An Introduction to Adaptive Arrays*, John Wiley, New York, 1980.

[10] J. H. Winters, "Optimum Combining in Digital Mobile Radio with Cochannel Interference," IEEE Trans. Veh. Tech., Vol 33, pp. 144-155, Aug. 1984.

[11] R. L. Cupo, et al. "A Four Element Adaptive Antenna Array for IS-136 PCS Base Stations," VTC 1997, pp. 1577-1581.

[12] G. D. Golden, C. C. Martin, N. R. Sollenberger and J. H. Winters, "Adaptive Communication System and Method Using Unequal Weighting of Interference and Noise," United States Patent No. 5,819,168.

[13] C. C Powell and J. Boccuzzi, "Performance of $\pi/4$-QPSK Baseband Differential Detection under Frequency Offset Conditions," IEEE Globecom, 1991.