Canonical Expressions for the Error Probability Performance of M-ary Modulation with Hybrid Selection/Maximal-Ratio Combining in Rayleigh Fading

Moe Z. Win†, *Senior Member, IEEE,* Ranjan K. Mallik, *Member, IEEE,*
George Chrisikos, *Member, IEEE,* and Jack H. Winters, *Fellow, IEEE*

Abstract— **Hybrid selection/maximal-ratio combining (H-S/MRC) is a reduced-complexity diversity combining scheme, where L out of N diversity branches (with the largest signal-to-noise ratio (SNR) at each instant) are selected and combined using maximal-ratio combining (MRC). In this paper, we derive *closed-form* expressions for the symbol error probability (SEP) of a H-S/MRC diversity system with arbitrary L and N. We consider coherent detection of M-ary phase-shift keying (MPSK) for the case of independent Rayleigh fading with equal SNR averaged over the fading.**

I. Introduction

The capacity of wireless systems in a multipath environment can be increased by diversity techniques [1], such as selection combining (SC) or maximal-ratio combining (MRC) [2]. SC is the simplest form of diversity combining whereby the received signal is selected from *one* out of N available diversity branches. In MRC, the received signals from *all* the diversity branches are weighted and combined to maximize the *instantaneous* signal-to-noise ratio (SNR) at the combiner output.

Though a high diversity order is possible in many situations, it may not be feasible to utilize all of the available branches. For example, a large order of antenna diversity may be obtained, particularly at higher frequencies such as the PCS bands, using spatial separation and/or orthogonal polarizations. Similarly, for spread spectrum receivers operating in dense multipath environments, the number of resolvable paths (or diversity branches) increases with the transmission bandwidth [3], [4]. However, the available correlator resources limit the number of paths that can be utilized in a typical Rake combiner [4].

This has motivated the study of diversity combining techniques that process only a *subset* of the available diversity branches with limited resources. One notable example is hybrid selection/maximal-ratio combining (H-S/MRC) which selects the L branches with the largest SNR at each instant, and then combines these branches to maximize the SNR. Recently, H-S/MRC has been considered as an efficient means to combat multipath fading [5], [6], [7], [8], [9]. In [6], the bit error rate (BER) performance of H-S/MRC with $L = 2$ and $L = 3$ out of N branches was analyzed, and it was pointed out that "the expressions become extremely unwieldy" for $L > 3$. The average SNR of H-S/MRC was derived in [7]. In [8], a "virtual branch" technique was introduced to succinctly derive the mean as well as the variance of the combiner output SNR of the H-S/MRC diversity system. In [9], we derived exact expressions for the SEP of H-S/MRC that involve a single integral with finite limits.

In this paper, we derive *closed-form* expressions for the symbol error probability (SEP) of a H-S/MRC diversity system with *arbitrary* L and N. We consider coherent detection of M-ary phase-shift keying (MPSK) for the case of independent Rayleigh fading with equal SNR averaged over the fading. We obtain a *canonical structure* for the SEP of H-S/MRC as a weighted sum of the elementary SEP's, which are the SEP's for MRC of independent and identically distributed branches.

II. Hybrid S/MRC Analysis via the Virtual Branch Technique

A. Hybrid-S/MRC

Let γ_i denote the instantaneous SNR of the i^{th} diversity branch defined by

$$\gamma_i \triangleq \alpha_i^2 \frac{E_s}{N_{0i}}, \tag{1}$$

where E_s is the average symbol energy, and α_i is the instantaneous fading amplitude and N_{0i} is the noise power spectral density of the i^{th} branch. We model the γ_i's as continuous random variables with probability density function (p.d.f.) $f_{\gamma_i}(x)$ and mean $\Gamma_i = \mathbb{E}\{\gamma_i\}$.

Moe Z. Win and Jack H. Winters are with Wireless Systems Research Department, Newman Springs Laboratory, AT&T Labs - Research, 100 Schulz Drive, Red Bank, NJ 07701-7033 USA (e-mail: win@research.att.com, jhw@research.att.com)

Ranjan K. Mallik is with the Department of Electrical Engineering, Indian Institute of Technology - Delhi, Hauz Khas, New Delhi 110016 India (e-mail: rkmallik@ee.iitd.ernet.in)

George Chrisikos is with Dot Wireless, Inc., 6825 Flanders Drive, San Diego, CA 92121-2905 USA (e-mail: gchrisikos@dotwireless.com).

†Corresponding author.

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APPENDICES

Appendix I: Decomposition Method

Let the function $f(x)$ be defined as

$$f(x) = \prod_{n=1}^{\tilde{N}} \left(\frac{c_n}{c_n + x} \right)^{\mu_n}, \tag{23}$$

where $\{-c_n\}$ are the $\tilde{N}$ distinct poles of $f(x)$, with each having algebraic multiplicity μ_n.

We can express $f(x)$ as a weighted sum of elementary functions of the form $\left(\frac{c_n}{c_n+x}\right)^k$, which results in the decomposition formula [17]

$$f(x) = \sum_{n=1}^{\tilde{N}} \sum_{k=1}^{\mu_n} W_{n,k} \left(\frac{c_n}{c_n + x} \right)^k. \tag{24}$$

The coefficients $W_{n,k}$ of the expansion for $n = 1, \ldots, \tilde{N}$, $k = 1, \ldots, \mu_n$ are given by

$$W_{n,k} = \frac{1}{c_n^k(\mu_n - k)!} \frac{d^{\mu_n - k}}{dx^{\mu_n - k}} \left\{ x^{\mu_n} f(x - c_n) \right\}\Big|_{x=0}. \tag{25}$$

By using *Faa di Bruno's* formula for the derivatives of a composite function as in [18], we can compute (25) as

$$W_{n,k} = \left(\prod_{\substack{p=1 \\ p \neq n}}^{\tilde{N}} \left(\frac{c_p}{c_p - c_n} \right)^{\mu_p} \right) \times \sum_{\substack{(l_1, \ldots, l_{\mu_n - k}) \\ 0 \le l_1, \ldots, l_{\mu_n - k} \le \mu_n - k \\ l_1 + 2l_2 + \cdots + (\mu_n - k) l_{\mu_n - k} = \mu_n - k}} \prod_{q=1}^{\mu_n - k} \frac{1}{l_q!} \left(\frac{1}{q} \sum_{\substack{p=1 \\ p \neq n}}^{\tilde{N}} \mu_p \left(\frac{c_n}{c_n - c_p} \right)^q \right)^{l_q}. \tag{26}$$

Appendix II: Evaluation of the Elementary SEP Integral $\mathcal{J}_{n,k}(c_n, \theta)$

By using the binomial theorem, the indefinite integral

$$\mathcal{J}_{n,k}(c_n, \theta) \triangleq \frac{1}{\pi} \int \left[\frac{\sin^2 \theta}{c_n^{-1} + \sin^2 \theta} \right]^k d\theta \tag{27}$$

can be shown to be equivalent to

$$\mathcal{J}_{n,k}(c_n, \theta) = \frac{1}{\pi}\theta + \sum_{i=1}^{k} (-1)^i \binom{k}{i} \mathcal{I}_{n,i}(c_n, \theta), \tag{28}$$

where

$$\mathcal{I}_{n,i}(c_n, \theta) \triangleq \frac{1}{\pi} \int \frac{1}{(1 + c_n \sin^2 \theta)^i} d\theta. \tag{29}$$

Letting $\tan\theta = \sqrt{\frac{1}{1+c_n}} \tan\phi$, then $\theta = g(\phi) \triangleq \tan^{-1}\left(\sqrt{\frac{1}{1+c_n}} \tan\phi\right)$. The Jacobian of this transformation is

$$J = \sqrt{1 + c_n} \left(\frac{\sec^2 \phi}{c_n + \sec^2 \phi} \right). \tag{30}$$

Using (30) along with algebraic manipulations, the integral in (29) can be written in terms of the variable ϕ as

$$\begin{aligned} \mathcal{I}_{n,i}(c_n, g(\phi)) &= \frac{1}{\pi} \frac{1}{(1 + c_n)^{i - \frac{1}{2}}} \int \left(1 + c_n \cos^2 \phi\right)^{i-1} d\phi \\ &= \frac{1}{\pi} \frac{1}{(1 + c_n)^{i - \frac{1}{2}}} \sum_{l=0}^{i-1} \binom{i-1}{l} c_n^l F_{2l}(\phi), \end{aligned} \tag{31}$$

where

$$F_{2l}(\phi) \triangleq \int \cos^{2l} \phi \, d\phi. \tag{32}$$

We now focus on the evaluation of the indefinite integral $F_{2l}(\phi)$. Note that $F_0(\phi) = \phi$, and integrating by parts using

$$F_{2l}(\phi) = \int \cos^{2l-1} \phi \cos \phi \, d\phi, \tag{33}$$

we obtain the recursion for $l = 1, 2, \ldots$ as

$$F_{2l}(\phi) = \frac{\cos^{2l-1} \phi \sin \phi}{2l} + \frac{(2l-1)}{2l} F_{2l-2}(\phi). \tag{34}$$

We can express (34) as

$$F_{2l}(\phi) = S_{2l-2}(\phi) + T_{2l-2} F_{2l-2}(\phi), \tag{35}$$

where

$$S_{2l-2}(\phi) = \frac{\cos^{2l-1} \phi \sin \phi}{2l}, \qquad l = 1, 2, \ldots \tag{36}$$

$$T_{2l-2} = \frac{(2l-1)}{2l}, \qquad l = 1, 2, \ldots \tag{37}$$

Solving the recursion in (35) yields

$$F_{2l}(\phi) = S_{2l-2}(\phi) + \sum_{p=1}^{l-1} \left[\prod_{q=p}^{l-1} T_{2q} \right] S_{2p-2}(\phi) + \left[\prod_{q=0}^{l-1} T_{2q} \right] \phi. \tag{38}$$

Substituting (36) and (37) into (38) gives

$$\begin{aligned} F_{2l}(\phi) &\triangleq \frac{(2l)!}{4^l [l!]^2} \left[\phi + \frac{\sin \phi}{2} \sum_{p=1}^{l} \frac{4^p [(p)!]^2}{(2p)!} \frac{\cos^{2p-1} \phi}{p} \right] \\ &= \frac{\binom{2l}{l}}{4^l} \left[\phi + \frac{\tan \phi}{2} \sum_{p=1}^{l} \frac{4^p}{\binom{2p}{p}} \frac{1}{p(1 + \tan^2 \phi)^p} \right]. \end{aligned} \tag{39}$$

Now by substituting $\phi = g^{-1}(\theta)$ we obtain,

$$\begin{aligned} \mathcal{I}_{n,i}(c_n, \theta) = {} & \frac{1}{\pi} \frac{1}{(1 + c_n)^{i - \frac{1}{2}}} \sum_{l=0}^{i-1} \binom{i-1}{l} \binom{2l}{l} \left(\frac{c_n}{4} \right)^l \\ & \times \left[\tan^{-1}\left(\sqrt{1 + c_n} \tan \theta\right) + \sqrt{1 + c_n} \frac{\tan \theta}{2} \right. \\ & \left. \times \sum_{p=1}^{l} \frac{4^p}{\binom{2p}{p}} \frac{1}{p\left(1 + (1 + c_n)\tan^2 \theta\right)^p} \right]. \end{aligned} \tag{40}$$

The instantaneous output SNR of the H-S/MRC system is given by

$$\gamma_{S/MRC} = \sum_{i=1}^{L} \gamma_{(i)} \qquad 1 \le L \le N\,, \tag{2}$$

where $\gamma_{(i)}$ is the ordered γ_i, i.e., $\gamma_{(1)} > \gamma_{(2)} > \ldots > \gamma_{(N)}$, and N is the number of available diversity branches [8]. It is apparent from (2) that SC and MRC are special cases of H-S/MRC. Note that the possibility of at least two equal $\gamma_{(i)}$ is excluded, since $\gamma_{(i)} \neq \gamma_{(j)}$ *almost surely* for continuous random variables γ_i.[1] We assume in this study that the γ_i's are independent with equal average SNR, i.e., $\Gamma_i = \Gamma$ for $i = 1, \ldots, N$.

B. Virtual Branch Transformation

It is important to note that the $\gamma_{(i)}$'s are *no* longer independent, even though the underlying γ_i's are independent. Hence, the analysis of H-S/MRC using the ordered physical branch variables $\gamma_{(i)}$'s is cumbersome and complicated. This is alleviated in [8] by transforming the instantaneous SNR of the ordered diversity branches, $\gamma_{(i)}$, into a new set of *virtual branch* instantaneous SNR's, V_n, using the following relation:

$$\gamma_{(i)} = \sum_{n=i}^{N} \frac{\Gamma}{n} V_n\,. \tag{3}$$

It can be verified that the instantaneous SNR's of the virtual branches are i.i.d. normalized exponential random variables. The key advantage of this formulation is that it allows the instantaneous output SNR for H-S/MRC to be expressed in terms of the i.i.d. virtual branch SNR variables as

$$\gamma_{S/MRC} = \sum_{n=1}^{N} b_n V_n\,, \tag{4}$$

where the coefficients b_n are given by

$$b_n = \begin{cases} \Gamma, & n \le L \\ \Gamma \frac{L}{n}, & \text{otherwise}\,. \end{cases} \tag{5}$$

C. Symbol Error Probability over the Channel Ensemble

The SEP for H-S/MRC in multipath-fading environments is obtained by averaging the conditional SEP over the channel ensemble as

$$P_{e,S/MRC} = \mathbb{E}_{\gamma_{S/MRC}}\{\Pr\{e|\gamma_{S/MRC}\}\} \tag{6}$$

where $\Pr\{e|\gamma_{S/MRC}\}$ is the *conditional* SEP, conditioned on the random variable $\gamma_{S/MRC}$. For coherent detection of M-ary phase-shift keying (MPSK), a representation for $\Pr\{e|\gamma_{S/MRC}\}$ involving a definite integral with *finite* limits is given by [12], [13], [14]

$$\Pr\{e|\gamma_{S/MRC}\} = \frac{1}{\pi}\int_0^{\Theta} e^{-\frac{c_{MPSK}}{\sin^2\theta}\gamma_{S/MRC}}\, d\theta\,, \tag{7}$$

where $c_{MPSK} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$.

Averaging $\Pr\{e|\gamma_{S/MRC}\}$ over the channel ensemble can be accomplished, using the technique of [15], [16], by substituting the expression for $\gamma_{S/MRC}$ directly in terms of the physical branch variables given in (2). Since the statistics of the ordered paths are *no* longer independent, the direct use of such a technique involves N-fold nested integrals, which are in general cumbersome and complicated to evaluate. This problem is alleviated using a virtual branch technique in [8] by expressing $\gamma_{S/MRC}$ in terms of the virtual branch variables via (3). In this framework, *exact* expressions for the SEP of H-S/MRC with *arbitrary* L and N in Rayleigh fading is derived in [9] as

$$P_{e,S/MRC} = \frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{c_{MPSK}\Gamma + \sin^2\theta}\right]^L \times \prod_{n=L+1}^{N}\left[\frac{\sin^2\theta}{c_{MPSK}\Gamma\frac{L}{n} + \sin^2\theta}\right] d\theta\,. \tag{8}$$

III. Closed-Form Evaluation of the SEP

The integral expression given in (8) requires numerical integration in order to evaluate the SEP performance. In this section, we provide a method to evaluate the integral in closed form as a canonical expression of finite weighted sums of elementary SEP's.

The SEP expression in (8) can be rewritten as

$$P_{e,S/MRC} = \frac{1}{\pi}\int_0^{\Theta}\prod_{n=1}^{\tilde{N}}\left[\frac{\sin^2\theta}{c_{MPSK}\Gamma\frac{L}{L+n-1} + \sin^2\theta}\right]^{\mu_n} d\theta\,, \tag{9}$$

where $\tilde{N} = N - L + 1$, and the μ_n's are given by

$$\mu_n = \begin{cases} L, & n = 1 \\ 1, & n = 2, \ldots, \tilde{N}\,. \end{cases} \tag{10}$$

Note that $\{c_{MPSK}\Gamma\frac{L}{L+n-1}\}$ are all distinct and each has algebraic multiplicity μ_n such that $\sum_{n=1}^{\tilde{N}}\mu_n = N$. Letting $x = \frac{1}{\sin^2\theta}$ and $c_n = \left(c_{MPSK}\Gamma\frac{L}{L+n-1}\right)^{-1}$, the integrand in (9) fits into the expression of (23) of Appendix I. Using the decomposition method given in (24) of Appendix I, (9) can be decomposed into

$$P_{e,S/MRC} = \sum_{k=1}^{L} W_{1,k}\,\mathcal{J}_{1,k}(c_1,\Theta) + \sum_{n=2}^{N-L+1} W_{n,1}\,\mathcal{J}_{n,1}(c_n,\Theta)\,, \tag{11}$$

where the expressions for the weighting coefficients $W_{n,k}$ and definite integral $\mathcal{J}_{n,k}(c_n,\Theta)$ are given below. Equation

[1] In our context, the notion of "almost sure" or "almost everywhere" can be stated mathematically as: if $\mathcal{N} = \{\gamma_{(i)} = \gamma_{(j)}\}$, then $\Pr\{\mathcal{N}\} = 0$ [10], [11].

(12) is in the form of a canonical expression as a weighted sum of the elementary SEP's.

With the aid of Appendix I, it can be shown that the weighting coefficients $W_{n,k}$ reduce to

$$W_{1,k} = (-L)^{L-k} \binom{N}{L} \sum_{\substack{(l_1,\ldots,l_{L-k}) \\ 0\le l_1,\ldots,l_{L-k}\le L-k \\ l_1+2l_2+\cdots+(L-k)l_{L-k}=L-k}} \prod_{q=1}^{L-k} \frac{1}{l_q!} \left(\frac{1}{q} \sum_{p=2}^{N-L+1} \frac{1}{(p-1)^q} \right)^{l_q}, \tag{12}$$

for $k = 1, \ldots, L$, and

$$W_{n,1} = \frac{(-1)^{L+n}}{(L+n-1)} \left(\frac{L}{n-1}\right)^L \frac{N!}{L!(n-2)!(N-L+1-n)!}, \tag{13}$$

for $n = 2, \ldots, N-L+1$.

The definite integral $\mathcal{J}_{n,k}(c_n, \Theta)$ is in the form of an elementary SEP and is given by

$$\mathcal{J}_{n,k}(c_n,\Theta) \triangleq \frac{1}{\pi} \int_0^{\Theta} \left[\frac{\sin^2\theta}{c_n^{-1} + \sin^2\theta} \right]^k d\theta. \tag{14}$$

The indefinite integral of the form in (14) is evaluated in Appendix II. After considering the integration limits, we obtain

$$\mathcal{J}_{n,k}(c_n,\Theta) = \frac{1}{\pi}\Theta + \sum_{i=1}^{k} (-1)^i \binom{k}{i} \mathcal{I}_{n,i}(c_n,\Theta), \tag{15}$$

where the definite integral $\mathcal{I}_{n,i}(c_n,\Theta)$ is given as

$$\begin{aligned} \mathcal{I}_{n,i}(c_n,\Theta) = & \frac{1}{\pi} \frac{1}{(1+c_n)^{i-\frac{1}{2}}} \sum_{l=0}^{i-1} \binom{i-1}{l} \binom{2l}{l} \left(\frac{c_n}{4}\right)^l \\ & \times \left[\tan^{-1}\left(\sqrt{1+c_n}\tan\Theta\right) + \sqrt{1+c_n}\frac{\tan\Theta}{2} \right. \\ & \left. \times \sum_{p=1}^{l} \frac{4^p}{\binom{2p}{p}} \frac{1}{p\left(1+(1+c_n)\tan^2\Theta\right)^p} \right]. \end{aligned} \tag{16}$$

A. Limiting Case 1: SC System

SC is the simplest form of diversity combining whereby the received signal from *one* of N diversity branches is selected [2]. The output SNR of SC is given by

$$\gamma_{\text{SC}} = \max_i \{\gamma_i\} = \gamma_{(1)}. \tag{17}$$

Note that SC is a limiting case of H-S/MRC with $L = 1$. Substituting $L = 1$ in (12), the SEP becomes

$$P_{e,\text{SC}} = \sum_{n=1}^{N} W_{n,1}\, \mathcal{J}_{n,1}(c_n,\Theta), \tag{18}$$

where $W_{n,1}$ and c_n for SC are given by

$$W_{n,1} = (-1)^{n+1}\binom{N}{n}, \qquad n = 1,\ldots,N, \tag{19}$$

and

$$c_n = \left(c_{\text{MPSK}}\Gamma\frac{1}{n}\right)^{-1}, \tag{20}$$

respectively.

B. Limiting Case 2: MRC System

In MRC, the received signals from *all* diversity branches are weighted and combined to maximize the SNR at the combiner output [2]. The output SNR of MRC is given by

$$\gamma_{\text{MRC}} = \sum_{i=1}^{N} \gamma_i = \sum_{i=1}^{N} \gamma_{(i)}, \tag{21}$$

Note that MRC is a limiting case of H-S/MRC with $L = N$. It is clear from (12) and (13) that in this case $W_{1,k} = 0$ for $k = 1, \ldots, N-1$, and $W_{1,N} = 1$. Therefore,

$$P_{e,\text{MRC}} = \mathcal{J}_{1,N}(c_1,\Theta) \tag{22}$$

where c_1 for MRC is given by $c_1 = (c_{\text{MPSK}}\Gamma)^{-1}$.

IV. Numerical Results

In this section, we illustrate the SEP results derived for H-S/MRC. The notation H-L/N is used to denote H-S/MRC that selects and combines L out of N branches. Note that H-1/1 is a single branch receiver, and H-1/N and H-N/N are N-branch SC and N-branch MRC, respectively.

Figure 1 shows the SEP for coherent detection of MPSK with M=4 (QPSK) versus the average SNR per branch for various L with $N = 8$. Note that SC and MRC upper and lower bound, respectively, the SEP for H-S/MRC. It is seen that most of the gain of H-S/MRC is achieved for small L, e.g., the SEP for H-S/MRC is within 1 dB of MRC when $L = N/2$.

Figure 2 shows the SEP for coherent detection of QPSK versus the average SNR per branch for various N with $L = 2$. Note that, although the incremental gain with each additional combined branch becomes smaller as N increases, the gain is still significant even with $N = 8$. Furthermore, for $L = 2$ at a 10^{-3} SEP, H-S/MRC with $N = 8$ requires about 10 dB lower SNR than 2-branch MRC.

V. Conclusions

We derived closed-form expressions of the symbol error probability (SEP) for coherent detection of MPSK with hybrid selection/maximal-ratio combining (H-S/MRC) in multipath-fading wireless environments. With H-S/MRC, L out of N diversity branches are selected and combined using maximal-ratio combining (MRC). This technique provides improved performance over L branch MRC when additional diversity is available.

We analyzed this system using a "virtual branch" technique which resulted in a simple derivation of the closed-form SEP expressions for *arbitrary* L and N. We further obtained a *canonical structure* for the SEP of H-S/MRC as a weighted sum of the elementary SEP's, which are the SEP's for MRC of independent and identically distributed branches. These closed-form expressions do not involve hypergeometric series or special functions and are valid for arbitrary modulation levels.

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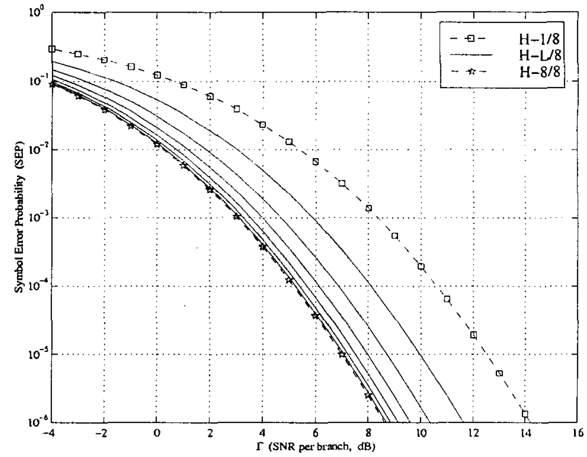

Fig. 1. The symbol error probability for coherent detection of QPSK with H-S/MRC as a function of the average SNR per branch in dB for various L with $N = 8$. The curves are parameterized by different L starting from the upper curve representing H-1/8, to the lowest curve representing H-8/8.

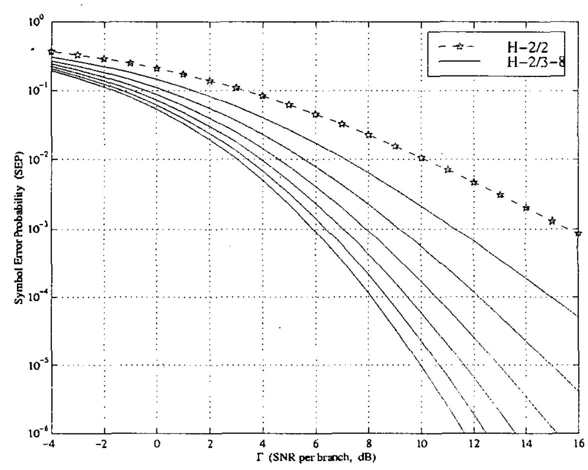

Fig. 2. The symbol error probability for coherent detection of QPSK with H-S/MRC as a function of the average SNR per branch in dB for various N with $L = 2$. The curves are parameterized by different N starting from the upper curve representing H-2/2, to the lowest curve representing H-2/8.