Error Probability for M-ary Modulation in Correlated Nakagami Channels using Maximal Ratio Combining

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***Abstract*—In this paper, we derive the symbol error probability for coherent detection of several types of M-ary modulation schemes using maximal ratio combining with an arbitrary number of diversity branches. We consider correlated Nakagami fading channels, where the instantaneous signal-to-noise ratios of the diversity branches are *not* necessarily independent *or* identically distributed. The proposed problem is made analytically tractable by transforming the correlated physical diversity branches into independent "virtual branches."**

I. Introduction

RECENTLY, Nakagami fading channels have received considerable attention in the study of various aspects of wireless systems [1], [2], [3], [4], [5], [6], [7], [8]. The Nakagami distribution, also known as the "m-distribution," provides greater flexibility in matching experimental data. Experimental results have shown that the Nakagami distribution fits experimental data collected in a variety of fading environments better than Rayleigh, Rician, or log-normal distributions [9], [10], [11]. The Nakagami family of distributions span from the one-sided Gaussian distribution (m=1/2) to the non-fading channel case ($m = \infty$), and contain Rayleigh fading (m=1) as a special case; along with the cases of fades that are more severe than Rayleigh ($1/2 \leq m < 1$) and fades that are less severe than Rayleigh ($1 < m$). They can also be used as an approximation to log-normal and Rician distributions for a certain range of average SNR's [3].

Analysis of MRC in Nakagami fading has typically assumed that diversity branches are independent [1], [6], [8]. Consideration of correlated fading has been limited only to dual-branch diversity [1], [12], with the exception of [7], [13]. The studies in [7], [13], though, consider only binary modulation with the assumption of equal m as well as average SNR's among all diversity branches. Furthermore, [7] assumed two specific correlation models, namely the equal-correlation and exponential-correlation models.

However, in some cases the average SNR is not necessarily equal for all the diversity branches and the fading statistics can also be different for each diversity branch. Examples include co-located antenna arrays, widely spatially-separated antennas of a macrodiversity system, frequency bins of a channelized receiver, or fingers of a Rake receiver in a wireless communication system, where MRC mitigates the effect of multipath fading. Some examples of unequal average SNR and/or fading statistics among all diversity branches are: 1) angle diversity using multiple beams where the average signal strength and fading statistics can be different in each beam; 2) polarization diversity with high base station antennas where for a vertically-polarized transmitter, the average received signal strength at the horizontally-polarized antenna is typically 6 to 10 dB lower than the vertically-polarized antenna; 3) macrodiversity, where the shadow fading is different at each antenna and different local scattering conditions can lead to different fading statistics; and 4) Rake receivers, where the distribution of signal power with delay is not uniform and the first arriving multipath component is more likely to be specular than the later components. In these cases, closed-form expressions for the performance of MRC are not previously available in the literature.

In this paper, we derive the exact SEP expressions for coherent detection of several types of M-ary modulation using MRC with an arbitrary number of diversity branches. We consider correlated Nakagami fading channels, where the instantaneous signal-to-noise ratios (SNR's) of the diversity branches are *not* necessarily independent *or* identically distributed. Specifically: 1) these SNR's can be arbitrarily correlated; 2) the SNR distributions can be from different Nakagami families, i.e., the fading parameters m's are not necessarily equal; and 3) the average SNR's (averaged over the fading) of the branches are not necessarily equal.

II. Diversity Combining Analysis

A. Preliminaries

With MRC, the received signals from multiple diversity branches are cophased, weighted, and combined to maximize the output SNR. The instantaneous output SNR with MRC is given by [14] as

$$\gamma_{\mathrm{MRC}} = \sum_{i=1}^{N} \gamma_i \,, \qquad (1)$$

where γ_i denotes the instantaneous SNR of the $i^{\underline{\text{th}}}$ diversity branch and N is the number of available diversity branches.

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The instantaneous SNR can be expressed by

$$\gamma_i \triangleq \alpha_i^2 \frac{E_s}{N_{0i}}, \quad (2)$$

where E_s is the average symbol energy, and α_i is the instantaneous fading amplitude and N_{0i} is the noise power spectral density of the i^{th} branch.

For a correlated Nakagami fading channel, the marginal p.d.f. of α_i is Nakagami distributed, and thus the marginal p.d.f. of γ_i is given by

$$g_{\gamma_i}(x; m_i/\Gamma_i, m_i) = \frac{1}{\Gamma(m_i)} \left(\frac{m_i}{\Gamma_i}\right)^{m_i} x^{m_i - 1} e^{-m_i x/\Gamma_i}, \quad (3)$$

where m_i denotes the Nakagami family and the SNR averaged over the fading in the i^{th} branch is $\Gamma_i = \mathbb{E}\{\gamma_i\}$. Note that the α_i's can be from different Nakagami families, where m_i and Γ_i are not necessarily equal among the branches. We will refer to (m_i, Γ_i) as Nakagami parameter pairs. As in [4], we assume that the m_i's are integers, noting that the measurement accuracy of the channel is typically only of integer order. Let $\gamma \triangleq (\gamma_1, \gamma_2, \dots, \gamma_N)$ and denote the joint p.d.f. of $\gamma_1, \gamma_2, \dots, \gamma_L$ by $f_\gamma(\{\gamma_i\}_{i=1}^N)$. In general,

$$f_\gamma(\{\gamma_i\}_{i=1}^N) \neq \prod_{i=1}^N g_{\gamma_i}(x; m_i/\Gamma_i, m_i) \quad (4)$$

since the γ_i's are correlated.

B. Virtual Branch Technique: The Key Idea

Conventional analysis of MRC in correlated Nakagami fading is, in general, cumbersome and complicated since: 1) the γ_i's are correlated; 2) the γ_i's can be from different Nakagami families where the m_i's are not necessarily equal; and 3) the Γ_i's are not necessarily equal.[1] The difficulty described above is alleviated in the following by transforming the dependent physical branch variables into a new set of independent *virtual branches* and expressing the combiner output SNR as a linear function of the independent virtual branch SNR's.

Let X_i be the $2m_i \times 1$ vector defined by

$$X_i \triangleq [X_{i,1}\ X_{i,2}\ \cdots\ X_{i,2m_i}]^t, \qquad i = 1, 2, \dots, N, \quad (5)$$

where $(\cdot)^t$ denotes transpose, and the elements $X_{i,k}$, are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance given by $\mathbb{E}\left\{X_{i,k}^2\right\} = \frac{\Gamma_i}{2m_i}$.

Let X be the $D_T \times 1$ vector defined by

$$X \triangleq \left[X_1^t\ X_2^t\ \cdots\ X_N^t\right]^t, \quad (6)$$

[1]Note that our model includes the case where only a *proper* subset of the branches have the same m_i's and/or Γ_i's. This is a subtle but important difference with previous studies where the analyses given in [1] and [7] required that the $\frac{m_i}{\Gamma_i}$'s are either *all* different or *all* equal.

where $D_T = \sum_{i=1}^N 2m_i$ is twice the sum of the Nakagami parameters.

By carefully constructing X, the statistical dependence among the N correlated branches can be related to the statistical dependence among the elements of X. When there is only second-order dependence, it suffices to construct the covariance matrix of X given by $K_X = \mathbb{E}\{XX^t\}$. Without loss of generality, one can assume that the γ_i's are indexed in increasing order of their Nakagami parameters, i.e., $m_1 \le m_2 \le \dots \le m_N$. We construct the correlation among the elements X such that

$$\mathbb{E}\{X_{i,k}X_{j,l}\} = \begin{cases} \frac{\Gamma_i}{2m_i} & \text{if } i = j \text{ and } k = l \\ \rho_{i,j}\sqrt{\frac{\Gamma_i}{2m_i}\frac{\Gamma_j}{2m_j}}, & \text{if } i \neq j \text{ but } \\ & k = l = 1, 2, \dots, 2\min\{m_i, m_j\} \\ 0, & \text{otherwise}. \end{cases} \quad (7)$$

This construction implies that the k^{th} entries of X_i and X_j, with $i \neq j$, are correlated for $k = 1, 2, \dots, 2\min\{m_i, m_j\}$. However, all the entries of X_i are mutually independent, and all other entries are independent. The relationship between the covariance of γ_i and γ_j and the covariance of the elements of X is given by:

$$\rho_{\gamma_i\gamma_j} \triangleq \frac{\mathbb{E}\{(\gamma_i - \mathbb{E}\{\gamma_i\})(\gamma_j - \mathbb{E}\{\gamma_j\})\}}{\sqrt{Var\{\gamma_i\}Var\{\gamma_j\}}} = \sqrt{\frac{\min\{m_i, m_j\}}{\max\{m_i, m_j\}}}\, \rho_{i,j}^2. \quad (8)$$

The lower and upper bounds for the correlation between the two Nakagami branches are given by $0 \le \rho_{\gamma_i\gamma_j} \le \sqrt{\frac{\min\{m_i,m_j\}}{\max\{m_i,m_j\}}}$.[2]

Let $\{\lambda_l\}$ be the set of L *distinct* eigenvalues of K_X where each λ_l has algebraic multiplicity μ_l such that $\sum_{l=1}^L \mu_l = D_T = \sum_{i=1}^N 2m_i$. By using the Karhunen-Loève (KL) expansion of the vector X as in [15], the combiner output SNR can be written as[3]

$$\gamma_{\text{MRC}} \stackrel{\text{L}}{=} \sum_{l=1}^L \lambda_l V_l, \quad (9)$$

where the notation "$\stackrel{\text{L}}{=}$" denotes "equal in their respective distributions" (or "equal in their respective Laws") [18], [19], [20], and the virtual branch variables V_l are *independent* chi-squared r.v.'s with μ_l degrees of freedom. Therefore the characteristic function (c.f.) of V_l is given by

$$\psi_{V_l}(j\nu) \triangleq \mathbb{E}\left\{e^{+j\nu V_l}\right\} = \left[\frac{1}{1 - 2j\nu}\right]^{\mu_l/2}. \quad (10)$$

[2]The fact that two Nakagami branches with *different* fading parameters m_i and m_j cannot be completely correlated (i.e., $\rho_{\gamma_i\gamma_j} < 1$) is not a drawback in our statistical representation, and it is just a manifestation of the basic fact that two r.v.'s with different distributions can not be completely correlated.

[3]A similar technique employing a frequency-domain KL expansion was used in [16] to study diversity combining in a frequency-selective Rayleigh-fading channel. Another technique similar to the KL expansion was also used in [17] to study the reception of noncoherent orthogonal signals in Rician and Rayleigh fading channels.

TABLE I

PARAMETERS FOR SPECIFIC MODULATION SCHEMES.

Modulation Scheme	K	$a_k(\theta)$	$\phi_k(\theta)$	Θ_k
MPSK	1	$\frac{1}{\pi}$	$\sin^2\left(\frac{\pi}{M}\right)\csc^2(\theta)$	$\pi\left(1-\frac{1}{M}\right)$
MQAM	2	$\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)$	$\frac{3}{2(M-1)}\csc^2(\theta)$	$\frac{\pi}{2}$
		$-\frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2$	$\frac{3}{2(M-1)}\csc^2(\theta)$	$\frac{\pi}{4}$
MPAM	1	$\frac{2}{\pi}\left(1-\frac{1}{M}\right)$	$\frac{3}{M^2-1}\csc^2(\theta)$	$\frac{\pi}{2}$
BFSK	1	$\frac{1}{\pi}$	$\frac{1}{2}\csc^2(\theta)$	$\frac{\pi}{2}$
BFSK_{min}	1	$\frac{1}{\pi}$	$\frac{1}{2}\left(1+\frac{2}{3\pi}\right)\csc^2(\theta)$	$\frac{\pi}{2}$
DE-BPSK	2	$\frac{2}{\pi}$	$\csc^2(\theta)$	$\frac{\pi}{2}$
		$-\frac{2}{\pi}$	$\csc^2(\theta)$	$\frac{\pi}{4}$
MSK	2	$\frac{2}{\pi}$	$\csc^2(\theta)$	$\frac{\pi}{2}$
		$-\frac{1}{\pi}$	$\csc^2(\theta)$	$\frac{\pi}{4}$

Denoting $f_{V_l}(\cdot)$ as the p.d.f. of V_l and $V \triangleq (V_1, V_2, \ldots, V_L)$, the joint p.d.f. of $V_1, V_2, \ldots, V_L$ is,

$$f_V(\{V_l\}_{l=1}^L) = \prod_{l=1}^{L} f_{V_l}(V_l). \tag{11}$$

III. SEP FOR M-ARY MODULATION OVER THE CHANNEL ENSEMBLE

The SEP for MRC in correlated Nakagami fading is obtained by averaging the conditional SEP over the channel ensemble,

$$P_e = \mathbb{E}_{\gamma_{\text{MRC}}}\left\{\Pr\left\{e|\gamma_{\text{MRC}}\right\}\right\}, \tag{12}$$

where $\Pr\{e|\gamma_{\text{MRC}}\}$ is the SEP conditioned on the random variable γ_{MRC}. For a general class of modulation schemes, $\Pr\{e|\gamma_{\text{MRC}}\}$ can be expressed as

$$\Pr\left\{e|\gamma_{\text{MRC}}\right\} = \sum_{k=1}^{K}\int_0^{\Theta_k} a_k(\theta)\, e^{-\phi_k(\theta)\,\gamma_{\text{MRC}}}\, d\theta, \tag{13}$$

where $a_k(\theta)$, $\phi_k(\theta)$ and Θ_k are parameters particular to the specific modulation format and are independent of γ_{MRC}. Table I lists these parameters for some common coherent modulations:[4] M-ary phase shift keying (MPSK), M-ary square quadrature amplitude modulation (MQAM) with $M = 2^l$ and l even, M-ary pulse amplitude modulation (MPAM), binary frequency shift keying (BFSK), BFSK with minimum correlation (BFSK_{min}), coherent detection of differentially encoded BPSK (DE-BPSK), and precoded minimum shift keying (MSK) [21], [22], [23].

[4]Since MRC requires channel phase estimates, it is generally used in conjunction with coherent modulation schemes. If channel phase estimates are not available, then one may resort to diversity combining techniques such as postdetection equal gain combining (EGC) with noncoherent or differentially coherent modulation.

Evaluation of the SEP can be accomplished using the techniques of [24], [25], by substituting the expression for γ_{MRC} directly in terms of the physical branch variables, which gives

$$P_e = \sum_{k=1}^{K}\int_0^{\Theta_k} a_k(\theta)\, \mathbb{E}_{\{\gamma_i\}}\left\{e^{-\phi_k(\theta)\sum_{i=1}^{N}\gamma_i}\right\} d\theta \tag{14}$$

$$= \sum_{k=1}^{K}\int_0^{\Theta_k} a_k(\theta) \int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} e^{-\phi_k(\theta)\sum_{i=1}^{N}\gamma_i} \times f_{\gamma_N}(\{\gamma_i\}_{i=1}^N)\, d\gamma_N \ldots d\gamma_2 d\gamma_1\, d\theta. \tag{15}$$

Note in (15) that, since the physical branches are *correlated*, direct use of the methods given in [24], [25] requires an N-fold integration for the expectation operation in (14). This can be alleviated by expressing γ_{MRC} in terms of the virtual branch variables using (9) as:

$$P_e = \sum_{k=1}^{K}\int_0^{\Theta_k} a_k(\theta)\, \mathbb{E}_{\{V_n\}}\left\{e^{-\phi_k(\theta)\sum_{l=1}^{L}\lambda_l V_l}\right\} d\theta$$

$$= \sum_{k=1}^{K}\int_0^{\Theta_k} a_k(\theta) \prod_{l=1}^{L} \psi_{V_l}\left(-\phi_k(\theta)\lambda_l\right) d\theta, \tag{16}$$

where we have used the fact that the V_l's are independent. The effectiveness of the virtual path technique is apparent by observing that the expectation operation in the above equation no longer requires an N-fold integration.

Substituting (10) into (16) gives

$$P_e = \sum_{k=1}^{K} \int_0^{\Theta_k} a_k(\theta) \prod_{l=1}^{L} \left[\frac{1}{1+2\phi_k(\theta)\lambda_l} \right]^{\mu_l/2} d\theta. \quad (17)$$

Thus the derivation of the SEP for the coherent detection of several types of M-ary modulation using N-branch MRC in correlated Nakagami fading reduces to a single integral over θ with finite limits, where the integrand is an L-fold product of a simple expression involving trigonometric functions with $L \leq N$.

IV. Special Cases

In this section, we study some special cases of the results obtained in the previous section.

A. Single-Branch Reception

Let us first consider the simplest possible case, namely single-branch reception (without diversity) in Nakagami fading with (m_1, Γ_1). For single-branch reception ($N = 1$), K_X is a $2m_1 \times 2m_1$ diagonal matrix having only one eigenvalue ($L = 1$) of multiplicity $\mu_1 = 2m_1$. The eigenvalue is $\lambda_1 = \mathbb{E}\{x_{1,j}^2\} = \frac{\Gamma_1}{2m_1}$. Substituting the appropriate parameters into (17), the SEP for coherent detection of M-ary modulation using a single-branch receiver in Nakagami fading reduces to

$$P_e = \sum_{k=1}^{K} \int_0^{\Theta_k} a_k(\theta) \left[\frac{1}{1+\phi_k(\theta)\frac{\Gamma_1}{m_1}} \right]^{m_1} d\theta. \quad (18)$$

Note that (18) is equivalent to the SEP for coherent detection of M-ary modulation in Rayleigh-fading channels using MRC with m_1 *identical* branches having *equal* SNR's of $\frac{\Gamma_1}{m_1}$.

B. Independent Nakagami Channels

B.1 Case 1: *All* Equal $\frac{\Gamma_i}{m_i}$

Here, we consider MRC with independent Nakagami fading with parameter pairs (m_i, Γ_i) such that $\frac{\Gamma_i}{m_i}$ are equal for *all* N branches. Note that this includes the case of $m_i = m$ and $\Gamma_i = \Gamma$. In this case, K_X is a $2mN \times 2mN$ diagonal matrix having only one eigenvalue ($L = 1$) of multiplicity $\mu_1 = 2mN$. The eigenvalue is $\lambda_1 = \frac{\Gamma}{2m}$, and

$$P_e = \sum_{k=1}^{K} \int_0^{\Theta_k} a_k(\theta) \left[\frac{1}{1+\phi_k(\theta)\frac{\Gamma}{m}} \right]^{mN} d\theta. \quad (19)$$

B.2 Case 2: *All* Distinct $\frac{\Gamma_i}{m_i}$

Here, we consider MRC with independent Nakagami fading with parameter pairs (m_i, Γ_i) such that $\frac{\Gamma_i}{m_i}$ are distinct for *all* N branches. Note that this includes the case for equal m_i but distinct Γ_i.[5] In this case, K_X has N distinct eigenvalues given by $\lambda_l = \frac{\Gamma_l}{2m_l}$, $l = 1, \dots, N$, each with multiplicity $\mu_l = 2m_l$. The SEP is given by

$$P_e = \sum_{k=1}^{K} \int_0^{\Theta_k} a_k(\theta) \prod_{l=1}^{N} \left[\frac{1}{1+\phi_k(\theta)\frac{\Gamma_l}{m_l}} \right]^{m_l} d\theta. \quad (20)$$

C. Correlated Nakagami Channels

C.1 Dual-branch diversity

Consider dual-branch MRC in Nakagami fading with correlation coefficient ρ and parameter pairs (m_1, Γ_1) and (m_2, Γ_2). In general, there are three distinct eigenvalues ($L = 3$) with multiplicities $\mu_1 = \mu_2 = 2m_1$ and $\mu_3 = 2(m_2 - m_1)$. Using these parameters, the SEP for coherent detection of M-ary modulation using dual-branch MRC in correlated fading is given by (17) with

$$\lambda_1 = \left(\frac{\Gamma_1}{2m_1} + \frac{\Gamma_2}{2m_2}\right) + \sqrt{\left(\frac{\Gamma_1}{2m_1} + \frac{\Gamma_2}{2m_2}\right)^2 - \frac{\Gamma_1}{m_1}\frac{\Gamma_2}{m_2}(1-\rho^2)},$$

$$\lambda_2 = \left(\frac{\Gamma_1}{2m_1} + \frac{\Gamma_2}{2m_2}\right) - \sqrt{\left(\frac{\Gamma_1}{2m_1} + \frac{\Gamma_2}{2m_2}\right)^2 - \frac{\Gamma_1}{m_1}\frac{\Gamma_2}{m_2}(1-\rho^2)}, \text{ and}$$

$$\lambda_3 = \frac{\Gamma_2}{2m_2}.$$

C.2 Identical Nakagami Channels with Arbitrary Correlation

Consider N branch MRC in Nakagami fading, with arbitrary correlation where the m_i's and Γ_i's are equal for all branches. This includes the two special cases considered in [7] with two specific correlation models, namely equal correlation and exponential correlation. In this case, the $\{\lambda_l\}$'s are the L *distinct* eigenvalues of the block-diagonal covariance matrix having $2m$ identical $N \times N$ diagonal blocks given by

$$K_{\tilde{Y}_i} = \frac{\Gamma}{2m} \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,N} \\ \rho_{1,2} & 1 & \cdots & \rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1,N} & \rho_{2,N} & \cdots & 1 \end{bmatrix}. \quad (21)$$

The SEP in correlated Nakagami fading can be obtained by using the appropriate eigenvalues and corresponding multiplicities in (17).

V. Numerical Results

In this section we graphically display the SEP results for coherent detection of QPSK using N-branch MRC in Nakagami fading. Figure 1 displays the case for i.i.d. Nakagami channels with equal m, as well as Γ, among all branches. It is apparent that the SEP performance improves with increasing m and N.

Figures 2 and 3 depict the case of identical Nakagami channels, but with exponential correlation. The parameters Γ and m are equal for each branch and the correlation matrix in (21) has elements $\rho_{i,j} = \rho^{|i-j|}$. It is noted that the SEP performance degrades as the correlation coefficient increases.

[5] This case was analyzed in [6] by approximating the sum of the squares of Nakagami r.v.'s by a single Nakagami r.v. with appropriate parameters. This approximation becomes exact when the Γ_i's are all equal.

In Fig. 4, the case of identical Nakagami channels with a tridiagonal correlation matrix is shown. The off-diagonal elements are all equal to zero except for the first off-diagonals $\rho_{1,2} = \rho$. The effect of the correlation is seen to be slightly less severe than for the exponential correlation model.

VI. Conclusions

We derived the SEP for coherent detection of several types of M-ary modulation using maximal ratio combining with an arbitrary number of diversity branches in correlated Nakagami fading channels, where the instantaneous SNR's of the diversity branches are *not* necessarily independent *or* identically distributed. A general expression was derived in terms of the parameters of the specific modulation scheme.

The proposed problem was made analytically tractable by transforming the physical diversity branches into the "virtual branch" domain. This work obviates the need for lengthy derivations in separate scenarios with different number of diversity branches and correlation models, as our results give a simple prescription for evaluating the exact SEP performance. Our results extend previously-derived results to cover numerous additional useful cases.

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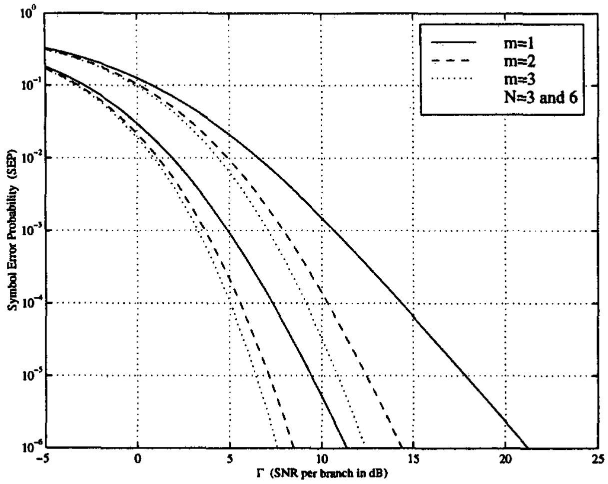

Fig. 1. The symbol error probability for QPSK with MRC as a function of the average SNR per branch in dB for i.i.d. Nakagami channels with parameter $m = 1, 2$ and 3. The number of diversity branches $N = 3$ for the upper set of curves and $N = 6$ for the lower set of curves.

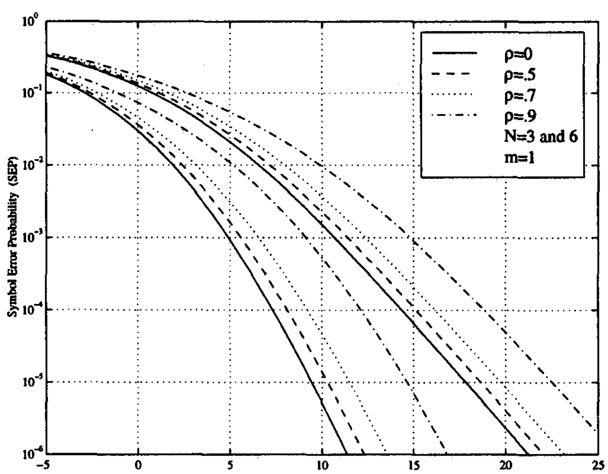

Fig. 2. The symbol error probability for QPSK with MRC as a function of the average SNR per branch in dB for identical Nakagami channels with exponential correlation. The correlation coefficient $\rho = 0, 0.5, 0.7$ and 0.9 with parameter $m = 1$. The number of diversity branches $N = 3$ for the upper set of curves and $N = 6$ for the lower set of curves.

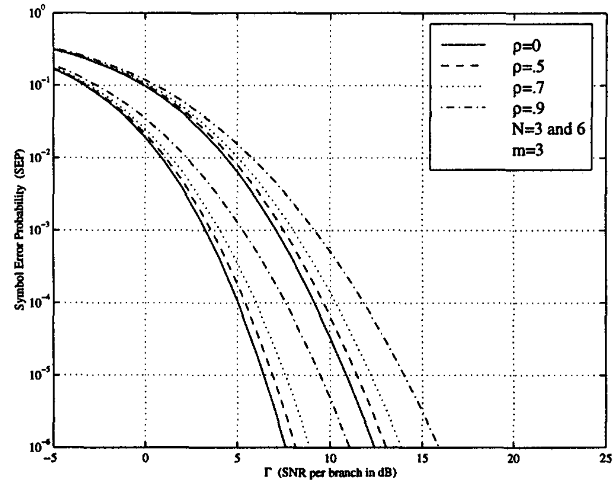

Fig. 3. The symbol error probability for QPSK with MRC as a function of the average SNR per branch in dB for identical Nakagami channels with exponential correlation. The correlation coefficient $\rho = 0, 0.5, 0.7$ and 0.9 with parameter $m = 3$. The number of diversity branches $N = 3$ for the upper set of curves and $N = 6$ for the lower set of curves.

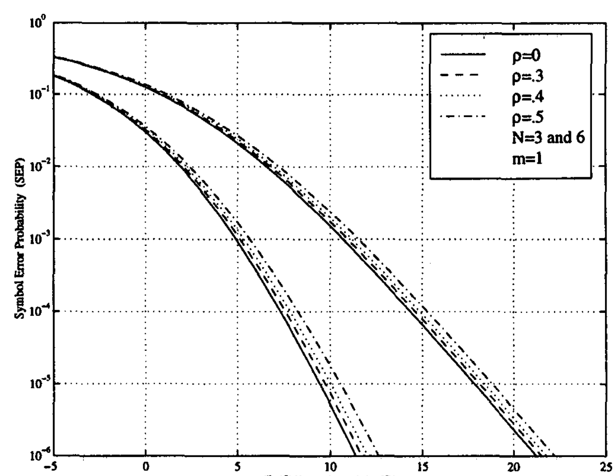

Fig. 4. The symbol error probability for QPSK with MRC as a function of the average SNR per branch in dB for identical Nakagami channels with tridiagonal correlation. The correlation coefficient $\rho = 0, 0.3, 0.4$ and 0.5 with parameter $m = 1$. The number of diversity branches $N = 3$ for the upper set of curves and $N = 6$ for the lower set of curves.