Performance of Predetection Dual Diversity in Correlated Rayleigh Fading: EGC and SD

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Abstract — **Using a series approach, expressions for the average symbol error probability (SEP) of coherent binary signals over a correlated Rayleigh fading channel with dual predetection equal gain combining (EGC) and selection diversity (SD) are derived. For both EGC and SD cases, the SEP is in terms of the correlation coefficient of the branch amplitudes, which is easy to compute and depicts clearly the effect of correlated fading on the error performance.**

I. Introduction

Previously published studies of the performance analysis of diversity combining in Rayleigh fading with both independent [1] [2] [3] and correlated diversity branches [4] [5] have focused mainly on maximal ratio combining (MRC) [3] [4] [5]. Equal gain combining (EGC) with coherent binary keying was studied with two and three independent branches in [6]. Analysis for selection diversity (SD) with two correlated branches was presented in [7], where the results were expressed in terms of finite integrals. In this paper we use a *series approach* instead of an integration approach to derive computationally simple expressions for the average symbol error probability (SEP) for coherent detection of binary signals with dual-diversity predetection EGC and SD. Both the cases of equal branch signal-to-noise-ratios (SNR's) and unequal branch SNR's are included in our framework.

II. Preliminaries

Let α_1 and α_2 be two correlated random variables which are marginally Rayleigh with second moments

$$\mathbf{E}\left\{\alpha_i^2\right\} = \Omega_i\,, \quad i = 1, 2, \tag{1a}$$

and *correlation coefficient*

$$\frac{\operatorname{cov}(\alpha_1^2, \alpha_2^2)}{\sqrt{\operatorname{var}(\alpha_1^2)\operatorname{var}(\alpha_2^2)}} = \rho, \quad 0 \le \rho < 1. \tag{1b}$$

The joint probability density function (p.d.f.) of α_1, α_2 is given by [8]

$$\begin{aligned} f_{\alpha_1,\alpha_2}(r_1, r_2) = {} & \tfrac{4 r_1 r_2}{\Omega_1\Omega_2(1-\rho)} \exp\left(-\tfrac{1}{(1-\rho)}\left[\tfrac{r_1^2}{\Omega_1} + \tfrac{r_2^2}{\Omega_2}\right]\right) \\ & \times I_0\left(\tfrac{2\rho^{\frac{1}{2}} r_1 r_2}{(1-\rho)(\Omega_1\Omega_2)^{\frac{1}{2}}}\right), \quad r_1, r_2 \ge 0. \end{aligned} \tag{2}$$

The joint cumulative distribution function (c.d.f.) of α_1, α_2 can be expressed in terms of the infinite series [8]

$$\begin{aligned} F_{\alpha_1,\alpha_2}(r_1, r_2) = {} & (1-\rho)\sum_{k=0}^{\infty} \tfrac{\rho^k}{(k!)^2}\, \Gamma\left(k+1,\, \tfrac{r_1^2}{\Omega_1(1-\rho)}\right) \\ & \times \Gamma\left(k+1,\, \tfrac{r_2^2}{\Omega_2(1-\rho)}\right), \quad r_1, r_2 \ge 0, \end{aligned} \tag{3}$$

where the incomplete gamma function $\Gamma(k+1, x)$ has the representation

$$\Gamma(k+1, x) = \int_0^x u^k e^{-u} du = k!\left[1 - e^{-x}\sum_{i=0}^{k} \frac{x^i}{i!}\right]. \tag{4}$$

In the performance analysis with EGC, we will use the joint characteristic function (c.f.) of α_1, α_2, which is given by [9] (p. 409)

$$\begin{aligned} \Psi_{\alpha_1,\alpha_2}(j\omega_1, j\omega_2) &= \mathbf{E}\left\{e^{j(\omega_1\alpha_1 + \omega_2\alpha_2)}\right\} \\ &= (1-\rho)\sum_{k=0}^{\infty} \frac{\rho^k}{(k!)^2} G_k(j\omega_1;\, \Omega_1, \rho) \\ &\quad \times G_k(j\omega_2;\, \Omega_2, \rho)\,, \end{aligned} \tag{5}$$

where, for $i = 1, 2$,

$$\begin{aligned} G_k(j\omega_i;\, \Omega_i, \rho) = {} & \Gamma(k+1)\,{}_1F_1\left(k+1;\, \tfrac{1}{2};\, -\tfrac{\omega_i^2\Omega_i(1-\rho)}{4}\right) \\ & + j\omega_i[\Omega_i(1-\rho)]^{\frac{1}{2}}\,\Gamma\left(k + \tfrac{3}{2}\right) \\ & \times {}_1F_1\left(k + \tfrac{3}{2};\, \tfrac{3}{2};\, -\tfrac{\omega_i^2\Omega_i(1-\rho)}{4}\right). \end{aligned} \tag{6}$$

Note that $\Gamma(\cdot)$ denotes the gamma function and ${}_1F_1(\cdot;\cdot;\cdot)$ denotes the confluent hypergeometric function.

For SD, we will use the c.f. of

$$\gamma_{\text{SD}} = \max\left\{\frac{\alpha_1^2}{c_1^2},\, \frac{\alpha_2^2}{c_2^2}\right\}, \tag{7}$$

where c_1 and c_2 are positive scale factors. Now the c.d.f. of γ_{SD} can be expressed as

$$F_{\gamma_{\text{SD}}}(v) = F_{\alpha_1,\alpha_2}\left(c_1 v^{\frac{1}{2}}, c_2 v^{\frac{1}{2}}\right), \tag{8}$$

and, using (3) and (4), its p.d.f. is given by

$$f_{\gamma_{\text{SD}}}(v) = \frac{d}{dv} F_{\alpha_1,\alpha_2}(c_1 v^{\frac{1}{2}}, c_2 v^{\frac{1}{2}})$$

$$= (1-\rho)\left[b_1 e^{-b_1(1-\rho)v} + b_2 e^{-b_2(1-\rho)v}\right] - (1-\rho)e^{-(b_1+b_2)v} \times \sum_{k=0}^{\infty} \frac{\rho^k}{k!} \sum_{i=0}^{k} \frac{(b_1^{k+1} b_2^i + b_2^{k+1} b_1^i)}{i!} v^{k+i}, \quad v \geq 0, \tag{9}$$

where

$$b_k = \frac{c_k^2}{\Omega_k(1-\rho)}, \quad k = 1, 2. \tag{10}$$

To obtain the c.f. of γ_{SD}, we use the result [10]

$$\int_0^{\infty} e^{j\omega v} e^{-av} v^k dv = \frac{k!}{(a - j\omega)^{k+1}}, \quad a > 0, \quad k = 0, 1, 2, \ldots$$

in (9). This gives

$$\Psi_{\gamma_{\text{SD}}}(j\omega) = (1-\rho)\left\{\frac{b_1}{[b_1(1-\rho) - j\omega]} + \frac{b_2}{[b_2(1-\rho) - j\omega]} - \sum_{k=0}^{\infty} \rho^k \sum_{i=0}^{k} \binom{k+i}{i} \frac{(b_1^{k+1} b_2^i + b_2^{k+1} b_1^i)}{[b_1 + b_2 - j\omega]^{k+i+1}}\right\}. \tag{11}$$

III. Equal Gain Combining

We consider a coherent dual-diversity reception system with a correlated flat Rayleigh fading channel, in which the receiver employs matched filter detection. With EGC, the received signals of each diversity branch are co-phased, combined, and coherently demodulated. The complex baseband signal received over the kth diversity branch in a symbol interval $0 \leq t < T_s$ can be represented as

$$r_k(t) = \alpha_k e^{-j\theta_k} s(t) + n_k(t), \quad k = 1, 2, \tag{12}$$

where $s(t)$ is the information-bearing signal, α_k and θ_k are the fading magnitude and phase of the kth diversity branch, and $n_k(t)$ represents the additive noise. The noises $n_1(t), n_2(t)$ are assumed to be independent zero-mean complex white Gaussian random processes with two-sided power spectral densities $2N_{01}$ and $2N_{02}$ respectively. We also assume independence among the random sequences $\{\alpha_k\}$, $\{\theta_k\}$ and $\{n_k(t)\}$. The fading magnitudes α_1, α_2 are assumed to be *correlated* Rayleigh random variables satisfying (1) with joint p.d.f. $f_{\alpha_1,\alpha_2}(\cdot,\cdot)$ given by (2).

We focus on the coherent detection of binary signals in which, over a symbol interval, $s(t) = s_i(t)$ if symbol i is transmitted, where $i = 0, 1$. The complex waveforms $s_0(t)$ and $s_1(t)$ have support $[0, T_s)$ and satisfy[1]

$$\begin{aligned} \textstyle\int_0^{T_s} |s_i(t)|^2 dt &= 2E_s, & i = 0, 1, \\ \Re\left\{\textstyle\int_0^{T_s} s_1(t) s_0^*(t) dt\right\} &= 2\epsilon E_s, & -1 \leq \epsilon < 1. \end{aligned} \tag{13}$$

[1]The notation $\Re\{\cdot\}$ stands for the real-part operator.

Note that ϵ is the *signal correlation coefficient* (correlation coefficient of $s_0(t)$ and $s_1(t)$). The decision rule of the receiver is given by

$$\Re\left\{\sum_{k=1}^{2} e^{+j\theta_k} \int_0^{T_s} r_k(t)[s_1^*(t) - s_0^*(t)]dt\right\} \underset{0}{\overset{1}{\gtrless}} 0. \tag{14}$$

This can be simplified using (13) to yield [6]

$$\begin{aligned} D_1 &\triangleq +(\alpha_1 + \alpha_2) + (W_1 + W_2) \underset{0}{\overset{1}{\gtrless}} 0 \quad \text{if symbol 1 is transmitted,} \\ D_0 &\triangleq -(\alpha_1 + \alpha_2) + (W_1 + W_2) \underset{0}{\overset{1}{\gtrless}} 0 \quad \text{if symbol 0 is transmitted,} \end{aligned} \tag{15a}$$

where

$$W_k = \frac{1}{2E_s(1-\epsilon)} \Re\left\{e^{j\theta_k} \int_0^{T_s} n_k(t)[s_1^*(t) - s_0^*(t)]dt\right\}, \quad k = 1, 2 \tag{15b}$$

are independent zero-mean real Gaussian random variables with variances

$$\mathbf{E}\left\{W_k^2\right\} = \frac{N_{0k}}{E_s(1-\epsilon)}, \quad k = 1, 2. \tag{15c}$$

By symmetry of $W_1 + W_2$, the average SEP is given by

$$P_{e,\text{EGC}} = \Pr(D_1 < 0) = \Pr(D_0 > 0). \tag{16}$$

If $F_{D_1}(\cdot)$ denotes the c.d.f. of the decision variable D_1 and $\Psi_{D_1}(j\omega)$ denotes its c.f., then, invoking the inversion theorem [11], we get from (16) [2]

$$P_{e,\text{EGC}} = F_{D_1}(0) = \frac{1}{2} - \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Im\{\Psi_{D_1}(j\omega)\}}{\omega} d\omega. \tag{17}$$

The c.f. of $D_1 = (\alpha_1 + \alpha_2) + (W_1 + W_2)$ is given by

$$\begin{aligned} \Psi_{D_1}(j\omega) &= \Psi_{W_1}(j\omega)\Psi_{W_2}(j\omega)\Psi_{\alpha_1+\alpha_2}(j\omega) \\ &= e^{-\frac{\omega^2(N_{01}+N_{02})}{2E_s(1-\epsilon)}} \Psi_{\alpha_1,\alpha_2}(j\omega, j\omega), \end{aligned} \tag{18}$$

where $\Psi_{\alpha_1,\alpha_2}(j\omega, j\omega)$ can be obtained from (5). Noting that

$$\Gamma(k+1) = k!, \quad \Gamma\left(k + \frac{3}{2}\right) = \frac{(2k+1)!}{k!\, 2^{2k+1}} \pi^{\frac{1}{2}},$$

[2]The notation $\Im\{\cdot\}$ stands for the imaginary-part operator.

we get from (18) and (5) the relation

$$\frac{\Im\{\Psi_{D_1}(j\omega)\}}{\omega} = e^{-\frac{\omega^2(N_{01}+N_{02})}{2E_s(1-\epsilon)}} \frac{(1-\rho)^{\frac{3}{2}}}{2\pi^{-\frac{1}{2}}} \sum_{k=0}^{\infty} \frac{(2k+1)!\,\rho^k}{(k!)^2 4^k} \times \left[\Omega_1^{\frac{1}{2}} g_k(\omega;\, \Omega_1[1-\rho], \Omega_2[1-\rho]) + \Omega_2^{\frac{1}{2}} g_k(\omega;\, \Omega_2[1-\rho], \Omega_1[1-\rho])\right], \quad (19a)$$

where

$$g_k(\omega;\, a_1, a_2) = {}_1F_1\left(k+\tfrac{3}{2};\, \tfrac{3}{2};\, -\tfrac{\omega^2 a_1}{4}\right) \times {}_1F_1\left(k+1;\, \tfrac{1}{2};\, -\tfrac{\omega^2 a_2}{4}\right). \quad (19b)$$

Now [9] (p. 1074)

$${}_1F_1\left(k+\tfrac{3}{2};\, \tfrac{3}{2};\, -x\right) = \frac{(-1)^k 2^{k+1}(k+1)!}{(2k+2)!} \times \frac{e^{-x}}{\sqrt{2x}} H_{2k+1}\left(\sqrt{2x}\right), \quad (20a)$$

where $H_{2k+1}(\cdot)$ is the Hermite polynomial of order $2k+1$ and is given by

$$H_{2k+1}(y) = (2k+1)! \sum_{m=0}^{k} \frac{(-1)^m}{m!\,(2k+1-2m)!\,2^m} y^{2k+1-2m}. \quad (20b)$$

Equations (20) and (19b) yield the result

$$g_k(\omega;\, a_1, a_2) = (-1)^k k!\, e^{-\frac{\omega^2 a_1}{4}} {}_1F_1\left(k+1;\, \frac{1}{2};\, -\frac{\omega^2 a_2}{4}\right) \times \sum_{m=0}^{k} \frac{(-1)^m}{m!\,(2k+1-2m)!} a_1^{k-m} \omega^{2k-2m}. \quad (21)$$

We next evaluate the integral

$$h_k(a_0, a_1, a_2) \triangleq \int_{-\infty}^{\infty} e^{-\frac{\omega^2 a_0}{4}} g_k(\omega;\, a_1, a_2) d\omega = (-1)^k k! \sum_{m=0}^{k} \frac{(-1)^m}{m!\,(2k+1-2m)!} a_1^{k-m} \times 2\int_0^{\infty} \omega^{2k-2m} e^{-\frac{\omega^2(a_1+a_0)}{4}} \times {}_1F_1\left(k+1;\, \frac{1}{2};\, -\frac{\omega^2 a_2}{4}\right) d\omega. \quad (22)$$

Changing the variable of integration to $u = \omega^2$ in (22), we obtain

$$h_k(a_0, a_1, a_2) = (-1)^k k! \sum_{m=0}^{k} \frac{(-1)^m}{m!\,(2k+1-2m)!} a_1^{k-m} \times \int_0^{\infty} u^{(k-m+\frac{1}{2})-1} e^{-\frac{u(a_1+a_0)}{4}} \times {}_1F_1\left(k+1;\, \frac{1}{2};\, -\frac{u a_2}{4}\right) du. \quad (23)$$

Using the result [12] (7.621(4))

$$\int_0^{\infty} u^{b-1} e^{-Su} {}_1F_1(a;\, c;\, -Ku) du = \Gamma(b)(S+K)^{-b} {}_2F_1\left(c-a,\, b;\, c;\, \frac{K}{S+K}\right), \quad b, S, K > 0$$

in (23), we get

$$h_k(a_0, a_1, a_2) = (-1)^k k! \sum_{m=0}^{k} \frac{(-1)^m}{m!\,(2k+1-2m)!} a_1^{k-m} \times \Gamma\left(k-m+\tfrac{1}{2}\right) \left(\tfrac{a_0+a_1+a_2}{4}\right)^{-(k-m+\frac{1}{2})} \times {}_2F_1\left(-k-\tfrac{1}{2},\, k-m+\tfrac{1}{2};\, \tfrac{1}{2};\, \tfrac{a_2}{a_0+a_1+a_2}\right), \quad (24)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes the Gaussian hypergeometric function [13]. Equations (19), (22) and (24) yield

$$\int_{-\infty}^{\infty} \frac{\Im\{\Psi_{D_1}(j\omega)\}}{\omega} d\omega = \frac{(1-\rho)^{\frac{3}{2}}}{2\pi^{-\frac{1}{2}}} \sum_{k=0}^{\infty} \frac{(2k+1)!\,\rho^k}{(k!)^2 4^k} \times \left[\Omega_1^{\frac{1}{2}} h_k\left(\tfrac{2(N_{01}+N_{02})}{E_s(1-\epsilon)},\, \Omega_1[1-\rho],\, \Omega_2[1-\rho]\right) + \Omega_2^{\frac{1}{2}} h_k\left(\tfrac{2(N_{01}+N_{02})}{E_s(1-\epsilon)},\, \Omega_2[1-\rho],\, \Omega_1[1-\rho]\right)\right]. \quad (25)$$

Let the branch SNR's be defined by

$$SNR_1 = \frac{\Omega_1 E_s}{\left(\frac{N_{01}+N_{02}}{2}\right)}, \quad SNR_2 = \frac{\Omega_2 E_s}{\left(\frac{N_{01}+N_{02}}{2}\right)}, \quad (26a)$$

and let

$$g = \frac{1-\epsilon}{2}. \quad (26b)$$

Using the fact that

$$\Gamma\left(k-m+\frac{1}{2}\right) = \frac{(2k-2m)!}{(k-m)!\,4^{k-m}} \pi^{\frac{1}{2}}$$

and changing the summation index m to $k-m$ in (24), we combine (25), (24) and (17) to obtain the final expression for the SEP, which is

$$P_{e,\text{EGC}} = \frac{1}{2} - \frac{(1-\rho)}{2} \sum_{k=0}^{\infty} \binom{2k}{k} \frac{\rho^k}{4^k} \sum_{m=0}^{k} \binom{k}{m} (-1)^m \left(\frac{2k+1}{2m+1}\right) \times \left[\left(\frac{SNR_1}{\frac{2}{g[1-\rho]}+SNR_1+SNR_2}\right)^{m+\frac{1}{2}} \times {}_2F_1\left(-k-\tfrac{1}{2},\, m+\tfrac{1}{2};\, \tfrac{1}{2};\, \frac{SNR_2}{\frac{2}{g[1-\rho]}+SNR_1+SNR_2}\right) + \left(\frac{SNR_2}{\frac{2}{g[1-\rho]}+SNR_1+SNR_2}\right)^{m+\frac{1}{2}} \times {}_2F_1\left(-k-\tfrac{1}{2},\, m+\tfrac{1}{2};\, \tfrac{1}{2};\, \frac{SNR_1}{\frac{2}{g[1-\rho]}+SNR_1+SNR_2}\right)\right] \quad (27)$$

The expression (27) is in terms of a series of powers of ρ, enabling easy computation of the SEP, and quantifying the effect of ρ on the SEP.

In the case of independent branches ($\rho = 0$), only the $k = 0$ term of the summation over k in (27) is non-zero. Using the fact that [13] (pp. 556–561)

$$ {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; \frac{1}{2}; x\right) = (1-x)^{\frac{1}{2}}, $$

(27) simplifies to

$$ P_{e,\mathrm{EGC}}|_{\rho=0} = \frac{1}{2} - \frac{\sqrt{SNR_1\left(SNR_1+\frac{2}{g}\right)} + \sqrt{SNR_2\left(SNR_2+\frac{2}{g}\right)}}{2\left(SNR_1+SNR_2+\frac{2}{g}\right)}, \quad (28) $$

which is the same as equation (23) of [6].

IV. Selection Diversity

In this case, the SEP, conditioned on the instantaneous output SNR γ_{SD}, for coherent binary keying can be expressed as

$$ P_{e,\mathrm{SD}}(\gamma_{\mathrm{SD}}) = Q\left(\sqrt{(1-\epsilon)\gamma_{\mathrm{SD}}}\right) = Q\left(\sqrt{2g\gamma_{\mathrm{SD}}}\right), \quad (29a) $$

where

$$ \gamma_{\mathrm{SD}} = \max\left\{\frac{E_s\alpha_1^2}{N_{01}}, \frac{E_s\alpha_2^2}{N_{02}}\right\}. \quad (29b) $$

The c.f. of γ_{SD} is given by (11) with

$$ b_k = \frac{N_{0k}}{\Omega_k E_s(1-\rho)}, \quad k = 1, 2. \quad (30) $$

The average SEP is given by

$$ \begin{aligned} P_{e,\mathrm{SD}} &= \mathbf{E}\left[P_{e,\mathrm{SD}}(\gamma)\right] \\ &= \int_0^\infty P_{e,\mathrm{SD}}(v) f_{\gamma_{\mathrm{SD}}}(v)\, dv. \quad (31) \end{aligned} $$

Using the technique of [14], the average SEP can be written in terms of the c.f. of γ_{SD} as

$$ P_{e,\mathrm{SD}} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Psi_{\gamma_{\mathrm{SD}}}\left(-\frac{g}{\sin^2\phi}\right) d\phi, \quad (32) $$

where $\Psi_{\gamma_{\mathrm{SD}}}(\cdot)$ is given by (11). We can therefore rewrite (32) as

$$ \begin{aligned} P_{e,\mathrm{SD}} = \frac{(1-\rho)}{\pi}\int_0^{\frac{\pi}{2}} \Bigg[\Bigg\{ &\frac{1}{(1-\rho)}\left(\frac{\sin^2\phi}{\frac{g}{b_1(1-\rho)} + \sin^2\phi}\right) \\ &+ \frac{1}{(1-\rho)}\left(\frac{\sin^2\phi}{\frac{g}{b_2(1-\rho)} + \sin^2\phi}\right) \\ &- \sum_{k=0}^{\infty}\rho^k \sum_{i=0}^{k} \binom{k+i}{i} \frac{(b_1^{k+1}b_2^i + b_2^{k+1}b_1^i)}{(b_1+b_2)^{k+i+1}} \\ &\times \left(\frac{\sin^2\phi}{\frac{g}{b_1+b_2} + \sin^2\phi}\right)^{k+i+1} \Bigg\} \Bigg] d\phi. \quad (33) \end{aligned} $$

The definite integral

$$ \mathcal{J}_k(a) \triangleq \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{a+\sin^2\phi}\right)^k d\phi, \quad a > 0, \quad k = 0, 1, 2, \ldots \quad (34) $$

can be evaluated to yield

$$ \begin{aligned} \mathcal{J}_k(a) = 1 &+ \sum_{l=1}^{k}(-1)^l \binom{k}{l} \left(\frac{a}{a+1}\right)^{l-\frac{1}{2}} \\ &\times \sum_{m=0}^{l-1} \binom{l-1}{m} \binom{2m}{m} \frac{1}{(4a)^m}. \quad (35) \end{aligned} $$

Applying (34) in (33), we get

$$ \begin{aligned} P_{e,\mathrm{SD}} = \frac{1}{2}&\left[\mathcal{J}_1\left(\frac{g}{b_1(1-\rho)}\right) + \mathcal{J}_1\left(\frac{g}{b_2(1-\rho)}\right)\right] \\ &- \frac{(1-\rho)}{2}\sum_{k=0}^{\infty}\rho^k \sum_{i=0}^{k}\binom{k+i}{i} \\ &\times \frac{(b_1^{k+1}b_2^i + b_2^{k+1}b_1^i)}{(b_1+b_2)^{k+i+1}} \mathcal{J}_{k+i+1}\left(\frac{g}{b_1+b_2}\right). \quad (36) \end{aligned} $$

Let the branch SNR's be defined in this case by

$$ SNR_1 = \frac{\Omega_1 E_s}{N_{01}}, \quad SNR_2 = \frac{\Omega_2 E_s}{N_{02}}, \quad (37) $$

On further simplification of (36) using (30) and (37), we get the final expression for the SEP, which is

$$ \begin{aligned} P_{e,\mathrm{SD}} = \frac{1}{2}&\left[2 - \sqrt{\frac{gSNR_1}{gSNR_1+1}} - \sqrt{\frac{gSNR_2}{gSNR_2+1}}\right] \\ &- \frac{(1-\rho)}{2}\sum_{k=0}^{\infty}\rho^k\sum_{i=0}^{k}\binom{k+i}{i} \\ &\times \frac{(SNR_1^i SNR_2^{k+1} + SNR_2^i SNR_1^{k+1})}{(SNR_1+SNR_2)^{k+i+1}} \\ &\times \mathcal{J}_{k+i+1}\left(\frac{gSNR_1 SNR_2(1-\rho)}{SNR_1+SNR_2}\right), \quad (38) \end{aligned} $$

where $\mathcal{J}_{k+i+1}(\cdot)$ is given by (35). Like the EGC expression (27), (38) is also in terms of a series of powers of ρ.

For independent branches ($\rho = 0$), only the $k = 0$ term of the summation over k in (38) is non-zero, and we get

$$ \begin{aligned} P_{e,\mathrm{SD}}|_{\rho=0} = \frac{1}{2}\Bigg\{1 &- \sqrt{\frac{gSNR_1}{gSNR_1+1}} - \sqrt{\frac{gSNR_2}{gSNR_2+1}} \\ &+ \sqrt{\frac{gSNR_1SNR_2}{gSNR_1SNR_2+SNR_1+SNR_2}}\Bigg\}, \quad (39) \end{aligned} $$

which is a known result [16].

V. Numerical Results

The SEP of BPSK, which corresponds to $g = 1$, is plotted against the branch SNR SNR_1 for EGC in Fig. 1 and for SD in Fig. 2, with different values of the correlation coefficient ρ ($\rho = 0, 0.5, 0.7, 0.9$). We have considered both the situations of equal branch SNR's ($SNR_1 = SNR_2$) and unequal branch SNR's ($SNR_1 > SNR_2$).

In the case of EGC, the Gaussian hypergeometric functions in (27) have been calculated using a truncated series formula with a relative error tolerance of 0.001. In computing $P_{e,\text{EGC}}$, the number of terms taken in the summation over k in (27) are: 15 for $\rho = 0.5$, 25 for $\rho = 0.7$, and 50 for $\rho = 0.9$. The maximum relative error obtained over all $P_{e,\text{EGC}}$ computations is 2.39 %. In the case of SD, the number of terms considered in the summation over k in (27) for computing $P_{e,\text{SD}}$ are: 15 for $\rho = 0.5$ and 25 for $\rho = 0.7, 0.9$, and the maximum relative error obtained over all $P_{e,\text{SD}}$ computations is 1.28 %.

The plots reveal that the SEP decreases with increase of branch SNR for a given ρ, and for given branch SNR's, the SEP increases with increase of ρ. In addition, the plots indicate that the performance with equal branch SNR's is better than that with unequal branch SNR's.

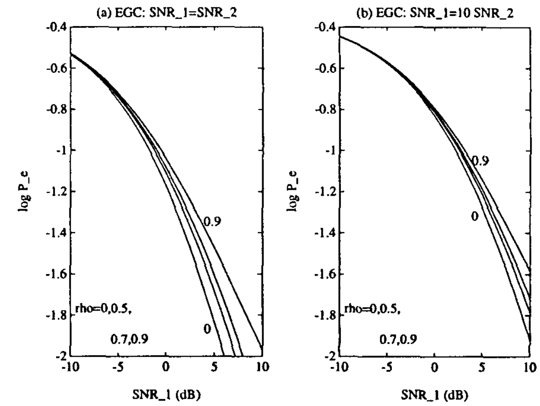

Fig. 1. Variation of $\log_{10} P_{e,\text{EGC}}$ with $10\log_{10} SNR_1$ when (a) $SNR_1 = SNR_2$, (b) $SNR_1 = 10\,SNR_2$ for BPSK having different values of the correlation coefficient ρ.

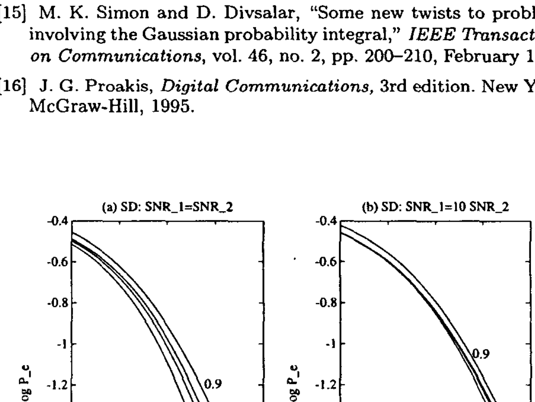

Fig. 2. Variation of $\log_{10} P_{e,\text{SD}}$ with $10\log_{10} SNR_1$ when (a) $SNR_1 = SNR_2$, (b) $SNR_1 = 10\,SNR_2$ for BPSK having different values of the correlation coefficient ρ.

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