Capacity of MIMO systems with antenna selection

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***Abstract*—We consider the capacity of multiple-input – multiple-output (MIMO) systems with reduced complexity. One link end uses all available antennas, while the other chooses the "best" L out of N antennas. As "best", we use those antennas that maximize capacity. We derive an upper bound on the capacity that can be expressed as the sum of the logarithms of ordered chi-squared variables. This bound is then evaluated analytically, and compared to results from Monte Carlo simulations. As long as L is at least as large as the number of antennas at the other link end, the achieved capacity is close to the capacity of a full-complexity system. We demonstrate, for example, that for $L = 3$, $N = 8$ at the receiver, and 3 antennas at the transmitter, the capacity of the reduced-complexity scheme is 20 bits/s/Hz compared to 23 bits/s/Hz of a full-complexity scheme.

I. Introduction

MIMO (multiple-input - multiple output) wireless systems are those that have antenna arrays at both transmitter and receiver. First simulation studies that reveal the potentially large capacities of those systems were already done in the 1980s [1], and a later paper explored the capacity analytically [2]. Since that time, interest in MIMO systems has exploded. Refs. [3], [4], and [5] gave guidelines for devising space-time (ST) codes that allow to approach the capacity limits revealed by [2]. Commercial products based on such codes are under development. Recent developments concentrate on finding improved codes [6], reduced-complexity codes [7], and the use of OFDM in MIMO systems [8], [9]. Most importantly, the standard for third-generation cellular phones (3GPP) foresees the use of a simple ST code [10] with two transmit antennas [11].

In earlier work, it was shown that the incremental gain of additional receive antennas is negligible if the total number of receive antennas N_r is far larger than the number of transmit antennas N_t [3].[1] This can be explained by the fact that additional antennas do not provide independent communications channels, but just increase the amount of diversity. However, a diversity order higher than, say, 3, does not significantly improve performance. This motivated us to explore the possibility of replacing the maximal-ratio-diversity that is normally achieved in a such a MIMO system with selection diversity. Thus, in this paper, we propose a reduced-complexity MIMO scheme that selects the L_r "best" of the available N_r antennas. This provides the full number of independent communications channels, and additionally a selection diversity gain. Compared to the use of all antennas, this has the advantage that only L_r instead of N_r receiver RF chains are required. We still require the full number of antenna elements, but these are usually inexpensive, as they are patch or dipole antennas that can be easily produced and placed.

For standard diversity reception, the principle of using L out of N antennas is known as "hybrid selection/maximum ratio combining" [12], [13], [14], [15]. Since we are using the selection (at one link end) to optimize the capacity of a MIMO system, we will refer to it as "hybrid selection/MIMO" (H-S/MIMO). In this paper, we are considering a system that uses all available antennas at one link end, while employing H-S/MIMO at the other link end. The case that both link ends use H-S/MIMO is treated in [16] using Monte Carlo simulations; this paper also develops a criterion for optimal antenna set selection. Reference [17] has shown that antenna selection is beneficial in a low-rank environment.

The rest of the paper is organized as follows: in Sec. II, we set up the system model and discuss the application for H-S/MIMO at one link end. Analytical bounds for the capacity are derived in Section III. Section IV gives evaluations and compares them to numerical simulation results. Conclusions and system design considerations are given in Section V.

II. System model

Figure 1 exhibits a block diagram of the considered system. For ease of notation, we always refer to the case where the transmitter uses all available antennas, while the receiver uses H-S/MIMO. At the transmitter, the data stream enters a space-time coder, whose output is forwarded to the N_t transmit antennas. The signals are subsequently upconverted to passband, amplified by a power amplifier, and filtered. In this study, we omit these stages, as well as their equivalents at the receiver, which allows us to treat the whole problem in equivalent baseband.

From the antennas, the signal is sent through the mobile radio channel, which is assumed to be flat-fading and quasi-stationary. Furthermore, the fading at the antennas is as-

[1] Under certain circumstances, increasing that number can even become harmful, as the channel estimation becomes more difficult and introduces estimation errors.

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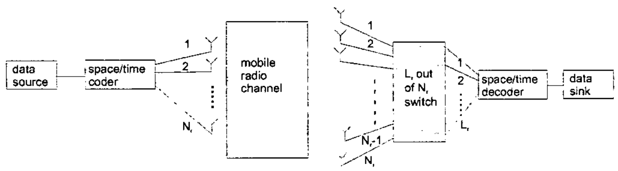

Fig. 1. Block diagram of the considered system.

sumed to be independent identically distributed (i.i.d.); for discussions of these assumptions, see, e.g., [18], [19], [20] and [21].

We denote the $N_t * N_r$ matrix of the channel as

$$H = \begin{pmatrix} h_{11} & h_{12} & . & . & . & h_{1N_r} \\ h_{21} & h_{22} & . & . & . & h_{2N_r} \\ . & . & . & . & . & . \\ h_{N_t1} & h_{N_t2} & & & & h_{N_tN_r} \end{pmatrix}. \quad (1)$$

If the channel is Rayleigh fading, the h_{ij} are i.i.d. zero-mean, circularly symmetric complex Gaussian random variables with unit variance, i.e. the real and imaginary part each have variance $1/2$. Consequently, the power carried by each transmission channel (h_{ij}) is chi-square distributed with 2 degrees of freedom. The channel also adds white Gaussian noise, which is assumed to be independent among the N_t receiver antenna elements.

The received signal is sent to the best L_r of the available N_r antenna elements (note that only L_r receiver chains are available). ST- coder and -decoder are assumed to be ideal so that the capacity can be achieved. However, we do *not* assume knowledge of the channel at the transmitter. Thus, the available transmitter power is distributed uniformly among all employed transmit antennas, since no waterfilling can be used.

III. Theory

A. Exact expression for the capacity

The capacity of MIMO system using all antenna elements is given by [2]

$$C_{\text{full}} = \log_2\left[\det\left(I_{N_t} + \frac{\overline{\Gamma}}{N_t}HH^\dagger\right)\right], \quad (2)$$

where I_{N_t} is the $N_t \times N_t$ identity matrix, $\overline{\Gamma}$ is the mean signal-to-noise ratio (SNR) per receiver branch, and superscript † denotes the Hermitian transpose. The receiver now selects those antennas that allow a maximization of the capacity, so that

$$C_{\text{select}} = \max_{S(\widetilde{H})}\left(\log_2\left[\det\left(I_{N_t} + \frac{\overline{\Gamma}}{N_t}\widetilde{H}\widetilde{H}^\dagger\right)\right]\right), \quad (3)$$

where $\widetilde{H}$ is created by striking $N_r - L_r$ columns from H, and $S(\widetilde{H})$ denotes the set of all possible $\widetilde{H}$, whose cardinality is $\binom{N_r}{L_r}$.

The choice of the optimum antennas requires knowledge of the complete channel matrix. This seems to necessitate the use of N_r RF chains, which defeats the purpose of having a low-complexity system. However, in a sufficiently slowly-changing environment, the antennas can be multiplexed to the L_r RF chains during the training bits - in other words, a chain is connected to the first antenna during the first part of the training sequence, then to the second antenna during the next part, and so on. At the end of the training sequence, we pick the best L_r antennas. Thus, we only need a few more training bits, and not more RF chains. Especially in high-data-rate systems, those additional training bits decrease the spectral efficiency in a negligible way.

B. Capacity bound

An exact analytical solution for C_{select} seems difficult. Thus, we derive analytical bounds in this subsection, and check them with Monte Carlo simulations in Sec. IV. Our starting point is the upper capacity bound for the full-complexity system with $N_t \leq N_r$ [2]

$$C_{\text{full}} \leq \sum_{i=1}^{N_t}\log_2\left(1 + \frac{\overline{\Gamma}}{N_t}\gamma_i\right) \quad (4)$$

where the γ_i are independent chi-square distributed random variables with $2N_r$ degrees of freedom. The equality applies in the "unrealistic case when each of the N_t transmittted components is received by a separate set of N_r antennas in a manner where each signal component is received with no interference from the others" [2].

We are now considering the case where we select the best L_r out of N_r receive antennas, and furthermore $L_r \leq N_t$. We thus have to exchange the role of transmitter and receiver, and select those antennas whose instantaneous realizations of γ_i are the largest. The capacity bound with antenna selection is thus

$$C_{\text{bound}} = \sum_{i=N_r-L_r+1}^{N_r}\log_2(1+\rho\gamma_{(i)}), \quad (5)$$

where $\rho = \overline{\Gamma}/N_t$, and the $\gamma_{(i)}$ are *ordered* chi-square distributed variables with $2N_t$ degrees of freedom, out of a set of N_r, with $\gamma_{(1)}$ signifying the smallest variable. The joint statistics of the ordered SNRs $\gamma_{(i)}$ can be shown to be [15]

$$p_{\gamma_{(i)}}(\gamma_{(1)},\gamma_{(2)},...\gamma_{(N_r)}) = N_r!\prod_{i=1}^{N_r}\frac{\gamma_{(i)}^{N_t-1}}{\Gamma(N_t)}\exp\left(-\gamma_{(i)}\right), \quad (6)$$

for $\gamma_{(1)} < \gamma_{(2)} < ...... < \gamma_{(N_r)}$ and 0 otherwise, where $\Gamma(\cdot)$ is Euler's Gamma function [22].

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Thus, the characteristic function of the capacity bound is

$$\Phi(j\nu) = \int_0^\infty \int_0^{\gamma(N_r)} \dots \int_0^{\gamma(2)} e^{-j\nu \sum_{i=N_r-L_r+1}^{N_r} \log_2(1+\rho\gamma_{(i)})} \cdot \tag{7}$$

$$\cdot p_{\gamma_{(i)}}(\gamma_{(1)}, \gamma_{(2)}, \dots \gamma_{(N_r)}) d\gamma_1 \dots\dots d\gamma_{(N_r-1)} d\gamma_{(N_r)}.$$

First, we perform the integrations over the $N_r - L_r$ discarded antennas. This results in an expression of the form

$$d^{(N_r-L_r)} + \left[\sum_{p=1}^{N_r-L_r} \exp\left(-b_p^{(N_r-L_r)} \gamma_{(N_r-L_r+1)}\right) \right. \tag{8}$$

$$\left. \sum_{k=0}^{(N_r-L_r-p+1)(N_t-1)} c_{p,k}^{(N_r-L_r)} \gamma_{(N_r-L_r+1)}^k \right].$$

The values of the coefficients b, d, c are computed via an iteration.We initialize with

$$\begin{aligned} d^{(0)} &= 1 \\ b_p^{(0)} &= 0 \\ c_{p,k}^{(0)} &= 0 \end{aligned} \tag{9}$$

and then perform $N_r - L_r$ iterations (so that $q = 0 \dots\dots N_r - L_r - 1$)

$$b_p^{(q+1)} = b_p^{(q)} + 1 \qquad \text{for} \qquad 1 \le p \le q \tag{10}$$

$$\widehat{c}_{p,k}^{(q)} = c_{p,k-(N_t-1)}^{(q)} \tag{11}$$

for $(q - p + 2)(N_t - 1) \ge k \ge (N_t - 1)$ and 0 otherwise,

$$d^{(q+1)} = d^{(q)}(N_t - 1)! + \sum_{p=1}^{q} \sum_{t=0}^{(q-p+2)(N_t-1)} \widehat{c}_{p,t}^{(q)} \frac{t!}{(b_p^{(q+1)})^{t+1}} \tag{12}$$

$$c_{p,k}^{(q+1)} = \begin{cases} -\sum_{t=0}^{(q-p+2)(N_t-1)-k} \frac{\widehat{c}_{p,k+t}^{(q)}}{(b_p^{(q+1)})^{N_t}} \frac{(k+t)!}{k!} \\ \text{for} \quad 1 \le p \le q \\ -\frac{d^{(q)}}{(b_p^{(q+1)})^{t+1}} \frac{(N_t-1)!}{k!} \\ \text{for} \quad p = q+1 \end{cases} . \tag{13}$$

For the next step of the iteration, it is advantageous to rewrite Eq. 8

$$\sum_{p=0}^{N_r-L_r} \exp\left(-b_p^{(N_r-L_r)} \gamma_{(N_r-L_r+1)}\right) \tag{14}$$

$$\sum_{k=0}^{(N_r-L_r-p+1)(N_t-1)} c_{p,k}^{(N_r-L_r)} \gamma_{(N_r-L_r+1)}^{k+\alpha^{(N_r-L_r)}}$$

so that

$$\begin{aligned} c_{0,0}^{(N_r-L_r)} &= d^{(N_r-L_r)} \\ c_{0,k}^{(N_r-L_r)} &= 0 \\ b_0^{(N_r-L_r)} &= 0 \\ \alpha^{(N_r-L_r)} &= 0 \end{aligned} \tag{15}$$

We then perform the next $L_r - 1$ integrations, which yield an expression of the form

$$\sum_{p=0}^{N_r-L_r} \exp\left(-\widehat{b}_p^{(N_r-1)} \gamma_{(N_r)}\right) \sum_{k=0}^{M} \widehat{c}_{p,k}^{(N_r-1)} \gamma_{(N_r)}^{k+\alpha^{(N_r-1)}} \tag{16}$$

where the parameters $\widehat{c}_{p,r}^{(N_r)}$, $\alpha^{(N_r)}$, and $\widetilde{b}_p^{(N_r)}$ are computed via a recursion. In each step, we first compute

$$\widehat{b}_p^{(q)} = b_p^{(q)} + 1 \tag{17}$$

$$\widehat{c}_{p,k}^{(q)} = \begin{cases} c_{p,k-(N_t-1)}^{(q)} & k = M \\ c_{p,k-(N_t-1)}^{(q)} + \frac{j\nu}{\rho \ln(2)} c_{p,k-N_t}^{(q)} & N_t - 1 \le k < M \\ \frac{j\nu}{\rho \ln(2)} c_{p,k-N_t}^{(q)} & k = N_t - 2 \\ 0 & \text{else} \end{cases} . \tag{18}$$

Then we can perform the second step, which is obtaining coefficients for the next iteration step

$$\alpha^{(q+1)} = \alpha^{(q)} + \frac{j\nu}{\ln(2)} \tag{19}$$

$$b_p^{(q+1)} = \widehat{b}_p^{(q)} \tag{20}$$

$$c_{p,r}^{(q+1)} = \sum_{k=0}^{r-1} \widehat{c}_{p,k}^{(q)} f_{p,r-1-k}^{(q)} \tag{21}$$

with

$$f_{p,n}^{(q)} = \frac{\left(\widehat{b}_p^{(q)}\right)^n}{\prod_{i=0}^{n} (k + \alpha^{(q)} + 1 + i)} . \tag{22}$$

The final integration and incorporation of constant multiplicative factors yields

$$\Phi(j\nu) = \frac{\rho^{j\nu L_r / \ln(2)} N_r!}{\Gamma(N_t)^{N_r}} \sum_{p=0}^{N_r-L_r} \sum_{r=0}^{M} \widehat{c}_{p,r}^{(N_r)} \frac{\Gamma(r + \alpha^{(N_r)} + 1)}{(\widetilde{b}_p^{(N_r)})^{r+\alpha^{(N_r)}+1}} . \tag{23}$$

The upper summation limit M is theoretically infinite, but the sum converges reasonably fast. In our computations,

$M = 50$ proved to be sufficient for $N_r = 8$. Details about the derivation of the recursion relations for the coefficients can be found in Ref. [23].

The above equation yields the characteristic function of the capacity bound (note that we have omitted the functional dependence of the parameters on ν for notational convenience). The pdf of the capacity bound is obtained by performing an inverse Fourier transformation (which can be accomplished by a Fast Fourier Transform FFT).

The bound derived above is quite tight for $L_r \leq N_t$, but tends to become rather loose for $L_r > N_t$. Especially, this bound suggests an "almost" linear increase of the capacity with L_r.[2] However, we have shown in Sec. II that we can only anticipate a logarithmic increase. A better upper bound for $L_r > N_t$ could be

$$C_{\text{bound}} = \sum_{i=1}^{N_t} \log_2(1 + \rho\tilde{\gamma}_{(i)}), \tag{24}$$

where the $\tilde{\gamma}_{(i)}$ are *ordered* chi-square distributed variables with $2L_r$ degrees of freedom, taken from a set of N_r available ones. The computation of this bound is a trivial variation of the method described in Sec. III.B.

IV. Results

In this section, we evaluate the bounds and compare them to computer experiments for practical system parameters. For the computer experiments (Monte Carlo simulations), we created random realizations of mobile radio channels. Each transfer function h_{ij} is an independent, identically distributed circularly complex Gaussian variable with zero mean and variance of $1/2$ for real and imaginary parts. Having thus created one realization of the matrix H, we created a complete set $S(\tilde{H})$ of the possible matrices $\tilde{H}$ by eliminating all possible permutations of $N_r - L_r$ rows from the matrix. For each of the $\tilde{H}$, we computed the capacity by Eq. 3, and selected the largest capacity from the set.

Figure 2 shows the cumulative distribution function (cdf) of capacity for $N_r = 8$, $N_t = 3$, and various L_r. The SNR is 20 dB, and in the following we consider the 10% outage capacity. With full exploitation of *all* available elements, 21.8 bit/s/Hz can be transmitted over the channel. This number decreases gradually as the number of selected elements L_r is decreased, reaching 18.2 bit/s/Hz at $L_r = 3$. For $L_r < N_t$, the capacity decreases drastically, since a sufficient number of antennas to provide N_t independent transmission channels is no longer available. These trends are well reflected in the bounds: the bound for the full-complexity system is 24.4 bit/s/Hz, decreasing to 20.1 bits/s/Hz at $L_r = 3$. We also found that the bounds become tighter as the SNR increases.

Figure 3 shows the influence of the SNR on the achieved results. We plot the improvement of the 10% outage capacity over a single-antenna system. We see that the capacity increase is very large at low SNRs (factor of 25 @ $SNR = 0dB$), while for high SNRs, it tends to a fixed value of about 4. A factor of 3 in the capacity increase can be attributed to the number of independent communications channels between transmitter and receiver. The remainder of the capacity increase is due to the diversity effect. Note also that Figure 3 plots the change in 10% outage capacity. If we were to consider the mean capacity, the influence of the SNR on the relative capacity increase would be significantly reduced. For standard $N_r = L_r = N_t$ systems, the relative mean capacity increase becomes practically independent of the SNR.

[2]Note that the increase is only "almost" linear because we are dealing with *ordered* stochastic variables. Thus, including more terms in the summation tends to give terms that have a lower SNR and thus a lower capacity.

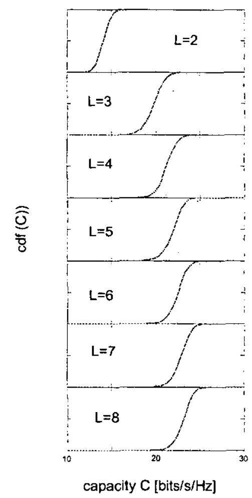

Fig. 2. Exact capacity (solid) and bound (dashed) for $N_r = 8$, $N_t = 3$, SNR= 20 dB.

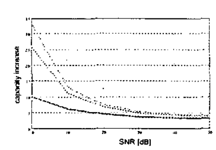

Fig. 3. Increase of 10% outage capacity of a system with $N_r = 8$, $L_r = 6$, $N_t = 3$, over a single-antenna system: bound (dashed), exact (dotted), and system with $N_t = L_r = 3$ (solid).

Another interesting point is the comparison between antenna selection criteria based on SNR, and those based on capacity. In our MC simulations, we also computed for each channel realization the antennas that should be selected from an SNR point of view. The indices of those antennas were then compared to the antennas that were chosen to maximize capacity. We found that only in about 50% of all channel realizations did the two selections agree with each other. This behavior can be interpreted in geometric terms by the insights of [18], which showed that for the deterministic case (corresponding to one channel realization), the phase shifts between the antenna elements are the decisive factors for capacity, and are far more important than instantaneous SNR. Figure 4 gives the capacities that are obtained by antenna selection based on an SNR criterion. We see that for $L_r \ll N_r$, the 10% outage capacity decreases from 18.0 to 14.3 bits/s/Hz @ 20 dB SNR when the SNR- instead of capacity- based criterion is used for antenna selection. This loss gets smaller as L_r approaches N_r.

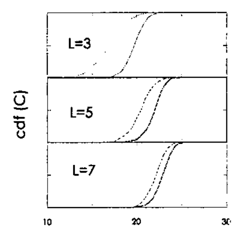

Fig. 4. Cdf of the capacity of a system with $N_r = 8$, $N_t = 3$. Selection of antenna by capacity criterion (solid) and by power criterion (dashed).

V. Summary and Conclusions

We have investigated the behavior of MIMO systems that select a subset of available antennas at one link end. Important applications for such systems are cellular systems with MIMO capability. The necessity of selecting antennas at one link end (instead of using all of them) stems either from complexity or cost considerations. For example, the number of different transmit antennas foreseen for the space/time coder could be limited as is already the case for the UMTS standard.

We derived upper bounds for the capacity of antenna selection, and compared them to results from computer simulations. The main result is that for $L_r \geq N_t$, selecting the best L_r antennas gives almost the same capacity as the full-complexity system. Capacity losses are less than 3.5 bits/s/Hz for $N_r = 8$, $N_t = 3$, at 20 dB SNR. This slight performance loss is offset by a considerable reduction in hardware costs. Instead of a full N_r transceiver chains, only L_r transceiver chains, plus an RF switch are required.

Thus, the results of this paper can serve as a guideline for designing reduced-complexity MIMO cellular systems for third- and fourth-generation communications.

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