Improved Space–Time Coding for MIMO-OFDM Wireless Communications

Rick S. Blum, Ye (Geoffrey) Li, Jack H. Winters, and Qing Yan

***Abstract*—Improved space–time coding for multiple-input and multiple-output orthogonal frequency division multiplexing is studied for wireless systems using QPSK modulation for four transmit and four receive antennas. A 256-state code is shown to perform within 3 dB of outage capacity (and within 2 dB with perfect channel estimation), which is better than any other published result without using iterative decoding.**

***Index Terms*—Antenna diversity, MIMO, OFDM, space–time codes.**

I. Introduction

THEORETICAL studies of communication links employing multiple transmit and receive antennas have shown great potential [1]–[4] for providing highly spectrally efficient wireless transmissions. The early investigations focused almost entirely on flat fading channels. Very recently [5], investigations have began to consider similar single-carrier approaches for frequency-selective fading channels with the hope of showing that similar gains could be achieved for mobile communications. These investigations are ultimately faced with a very complex equalization problem.

Here we consider an alternative approach, which employs multiple transmit and receive antennas in an orthogonal frequency division multiplexing (OFDM) communication system to produce what has been called a multiple-input and multiple-output (MIMO) OFDM system [6]. MIMO-OFDM greatly lessens, and possibly eliminates, the equalization complexity problem to produce an approach with tremendous potential. Very few investigations on this topic have appeared to date [6]–[8], and these investigations have not considered some promising MIMO-OFDM alternative approaches, as we attempt to demonstrate here. As has become common, we compare the performance of our approaches to the outage capacity. Comparisons of this type were not given in previous investigations of MIMO-OFDM.

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II. MIMO-OFDM and Space–Time Coding

Consider an OFDM communication system using n_t transmit antennas and n_r receive antennas. Such a system could be implemented using a single space–time encoder employing a code for n_t transmit antennas. In this case, the space–time encoder takes a single stream of binary input data and transforms it into n_t parallel streams of baseband constellation symbols. Each stream is broken into OFDM blocks with the nth block for the ith stream denoted by $t_i[n, k]$, $k = 0, \ldots, K-1$. Each OFDM block of constellation symbols is transformed using an inverse fast Fourier transform (IFFT) and transmitted by the antenna for its corresponding stream. Thus, all n_t transmit antennas simultaneously transmit the transformed symbols. The received signals at each antenna are similarly broken into blocks and processed using an FFT. After FFT processing, the nth block at receive antenna j is denoted by $r_j[n, k]$, $k = 0, \ldots, K-1$. At the receiver, a single space–time decoder employs a maximum likelihood sequence estimation (MLSE) algorithm to jointly decode the data blocks based on the observations from the n_r receive antennas.

Alternatively [8], we could employ n_g individual space–time encoders, where each encoder is designed to use n_t/n_g transmit antennas, as illustrated in Fig. 1 for the case of $n_t = 4$ and $n_g = 2$. In this case, the input to the pair of space–time encoders is divided into two streams, one for each encoder. At the receiver, an interference cancellation scheme is implemented by a space–time processor. The interference cancellation scheme attempts to separate the received signal due to one of the space–time encoders from the received signal due to the other space–time encoder. After this cancellation, again MLSE decoding is employed, followed by successive interference cancellation. We call the class of systems just described (for any n_t, n_r) a MIMO-OFDM system since in each case the overall channel can be viewed as a MIMO system due to the multiple transmit and receive antennas.

In either case, assuming proper cyclic extension and sample timing as well as tolerable leakage [9]

$$r_j[n, k] = \sum_{i=1}^{n_t} H_{ij}[n, k] t_i[n, k] + w_j[n, k] \qquad (1)$$

where $H_{ij}[n, k]$ denotes the normalized channel frequency response for the kth tone and OFDM block n, corresponding to the channel between the ith transmit antenna and the jth receive antenna. The normalization is such that $E\{|H_{ij}[n, k]|^2\} = 1$ and in fact we assume identical marginal statistics for each $H_{ij}[n, k]$, for all values of i, j, n, k. For convenience, we take $E\{|t_i[n, k]|^2\} = \rho_k$, $i = 1, \ldots, n_t$ so the transmitted power

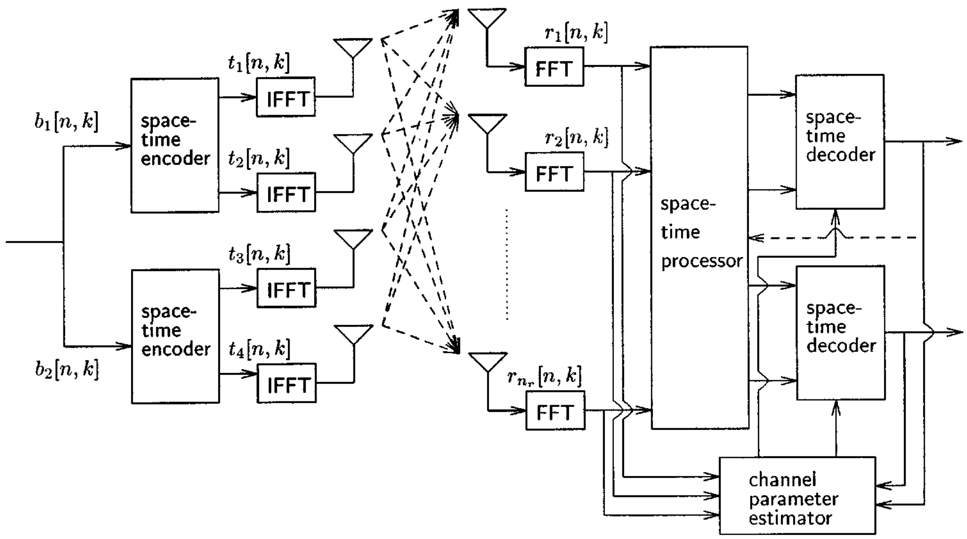


Fig. 1. MIMO-OFDM using $n_g = 2$ individual space–time encoders, each using $n_t/n_g = 2$ transmit antennas.

is the same from each antenna, which is reasonable in cases without feedback of channel state information to the transmitter, which are the cases we consider. In (1), $w_j[n, k]$ denotes the additive zero-mean, unit-variance complex Gaussian noise observed at the jth receive antenna for the kth tone of OFDM block n. Stacking the equations in (1) to obtain an equation for

$$\mathbf{r}[n, k] = (\, r_1[n, k] \quad \cdots \quad r_{n_r}[n, k]\,)^T \tag{2}$$

and using matrix multiplication to represent the sum gives the vector equation

$$\mathbf{r}[n, k] = \mathbf{H}[n, k]\mathbf{t}[n, k] + \mathbf{w}[n, k] \tag{3}$$

where $\mathbf{w}[n, k]$ has covariance matrix $\mathbf{I}_{n_r}$, which denotes an $n_r \times n_r$ identity matrix.

If the approach using the n_t-antenna space–time code is employed, then the MLSE algorithm chooses $\hat{\mathbf{t}}[n, k]$, its estimate of the transmitted signal, based on the metric

$$||\mathbf{r}[n, k] - \hat{\mathbf{H}}[n, k]\hat{\mathbf{t}}[n, k]||^2 \tag{4}$$

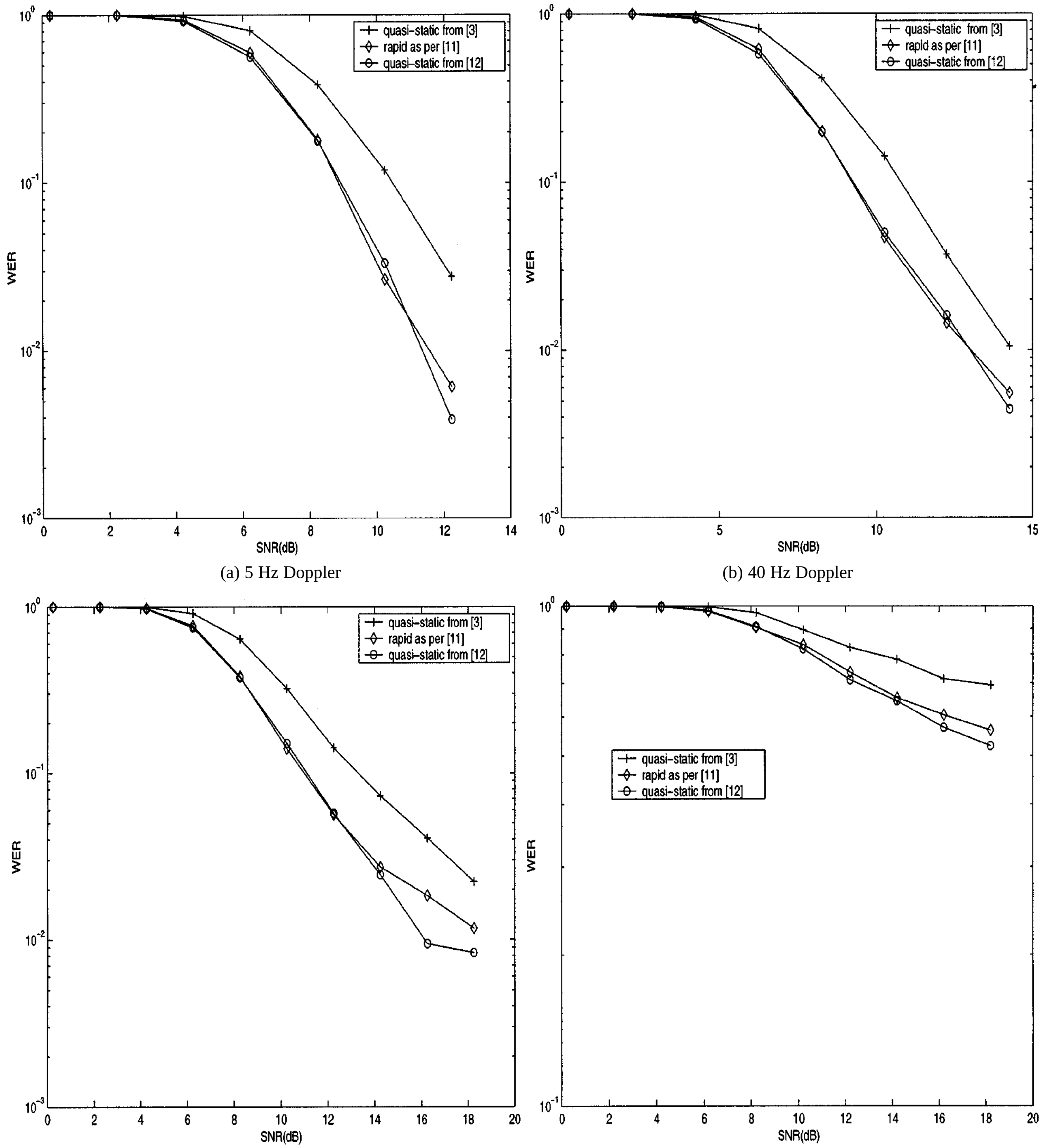
where $||\cdot||^2$ denotes the Euclidean norm and $\hat{\mathbf{H}}[n, k]$ denotes the estimate of $\mathbf{H}[n, k]$ from (3). An efficient signal detection approach for the system in Fig. 1 is provided in [8]. In this approach, the other space–time code is approximated as Gaussian interference, characterized by the instantaneous channel frequency response. This leads to a maximum-likelihood decoding approach which corresponds to first prewhitening the interference and then using an MLSE algorithm on the prewhitened observations.

III. Performance Evaluation

Now we present the performance of some MIMO-OFDM implementations with $n_t = n_r = 4$ assuming the Jakes fading model, the channel estimation procedures in [7] and [10] and the TU channel model considered in [7]. Our OFDM signals assume a channel bandwidth of 1.25 MHz, which is divided into 256 subchannels. Two subchannels at each end of the band are used as guard tones, with the other 252 tones used to transmit data. The symbol duration is taken to be 204.8 μs so that the tones are orthogonal. A 20.2-μs guard interval is used to provide protection from intersymbol interference, making the block duration $T_f = 225$ μs. The subchannel symbol rate is $r_b = 4.44$ kbaud. The parameters are chosen to be the same as those used in [8] for comparison.

First we consider the $n_g = 2$ MIMO-OFDM implementation proposed in [8] and illustrated in Fig. 1. In this case, two antenna space–time codes are employed that use 16 states and QPSK modulation. Data is grouped into blocks of 500 information bits, called words. Each word is coded into 252 symbols to form an OFDM block. Since this system uses $n_g = 2$, it can transmit two of these data blocks (1000 bits total) in parallel. Each time slot consists of 10 OFDM blocks with the first block used for training and the following 9 blocks used for data transmission. This leads to a system capable of transmitting 4 Mbit/s using 1.25 MHz of bandwidth, so the transmission efficiency is 3.2 bit/s/Hz.

In [8], an initial study of the system just outlined was provided. Several interference cancellation approaches were described and performance was evaluated. Here we focus on the interference cancellation approach based on signal quality and we assume that the same interleaving used in [8] will be employed. In [8], the space–time code used was the two-antenna, 16-state code given in [3, Fig. 5]. The word error rate (WER) achieved using this code is given in Fig. 2 for the case where the channel has a TU delay profile and for Doppler frequencies of 5, 40, 100, and 200 Hz. The other two curves in Fig. 2 illustrate the performance improvement that can be obtained using the improved space–time codes given in [11], [12]. One of these codes was designed to be optimum for the quasi-static fading model in [3]. The other code was designed to be optimum for the rapid fading model in [3]. The new improved codes from [11], [12] are optimum codes based on the criterion given in [3].



(a) 5 Hz Doppler (b) 40 Hz Doppler

(c) 100 Hz Doppler (d) 200 Hz Doppler

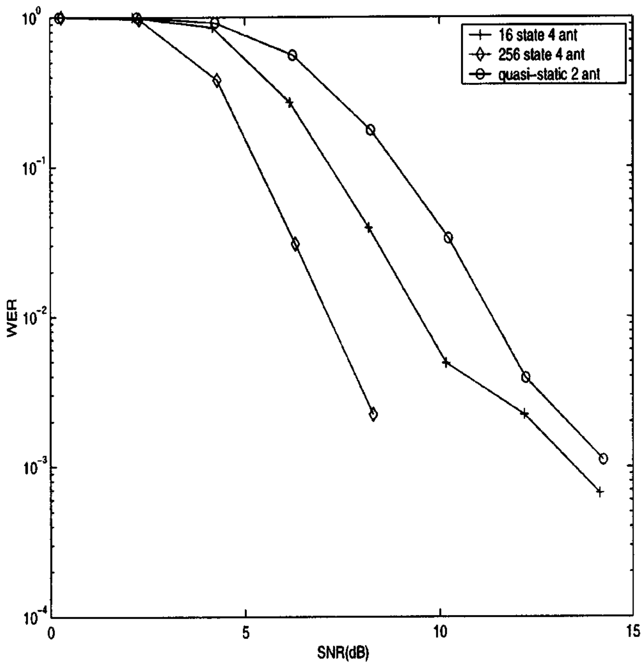
Fig. 2. WER versus SNR of MIMO-OFDM systems with $n_t = n_r = 4$, TU channel with different Doppler frequencies. See Table I for details on codes.

The improved codes produce roughly a 2-dB gain for the 5-Hz Doppler case (at WER $= 10^{-1}$). For larger Doppler frequencies, Fig. 2 shows that the gain is even larger. The two optimum codes appear to give about the same performance. This is not unreasonable since the channel model in this case includes aspects of both the quasi-static and rapid models in [3]. Also, these particular codes are known to be somewhat robust to mismatches in channel model.

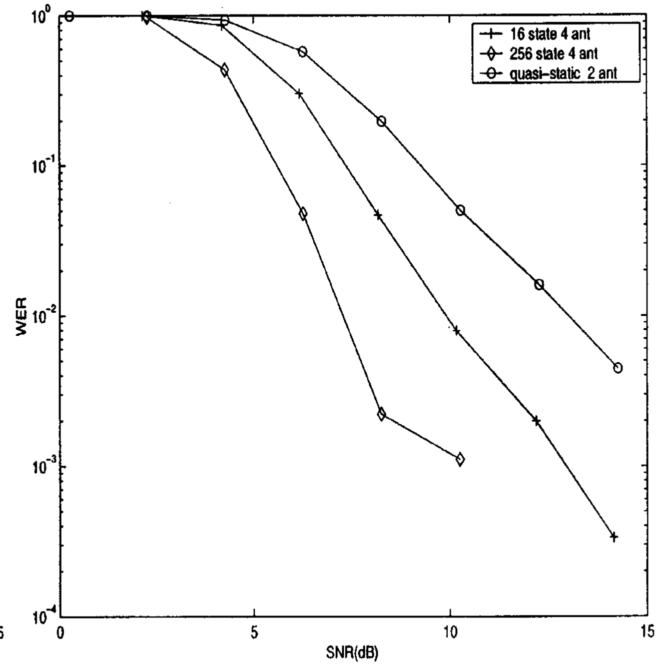
Next we investigate the approach that uses 4-antenna space–time codes. We consider 16-state and 256-state codes, designed using an *ad hoc* approach. Still, the performance of these codes

TABLE I
GENERATOR MATRICES FOR THE TWO TRANSMIT ANTENNA CODES USED IN FIGS. 2–4

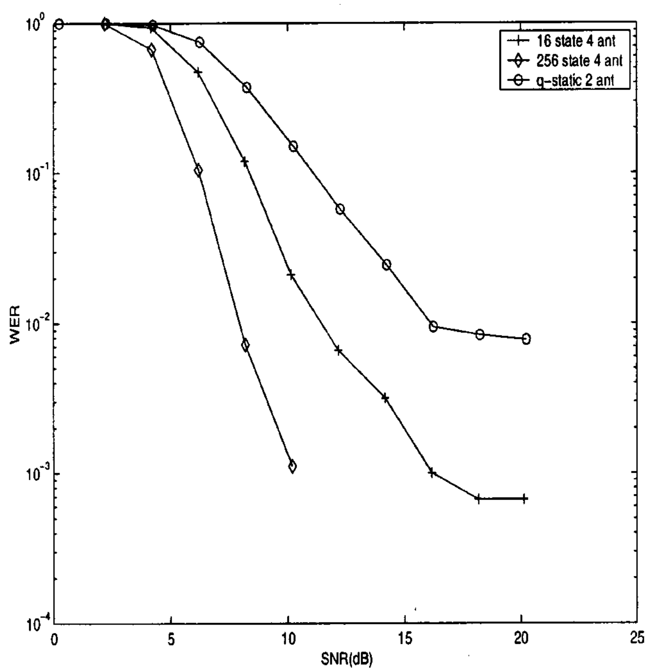
$\begin{pmatrix} D+2D^2 & 1+2D^2 \\ 2D & 2 \end{pmatrix}$	for the code from Figure 5 of [3]
$\begin{pmatrix} D+D^2 & 2+D \\ 2+D & 2+2D+2D^2 \end{pmatrix}$	for the quasi-static fading code from [11, 12]
$\begin{pmatrix} 2D^2 & 2+D+2D^2 \\ 2+D & 2D+2D^2 \end{pmatrix}$	for the rapid fading code from [11, 12].



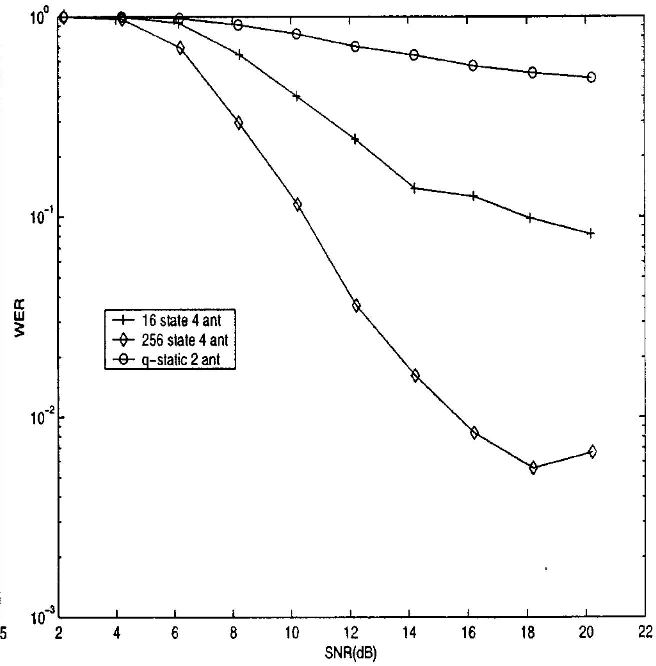
(a) 5 Hz Doppler



(b) 40 Hz Doppler



(c) 100 Hz Doppler



(d) 200 Hz Doppler

Fig. 3. WER versus SNR of MIMO-OFDM systems with $n_t = n_r = 4$, TU channel with different Doppler frequencies. Here we compare the best code from the last figure with codes designed for four transmit antenna cases. See Tables I and II for details on the codes.

is quite good when compared to the performance of the codes in Fig. 2. The comparisons are shown in Fig. 3 for the same cases considered in Fig. 2. The top curve, with the worst performance, is for the best scheme shown in Fig. 2. The middle curve is for the 16-state, 4-antenna space–time code. Note that this approach is better than the best approach from Fig. 2. The complexity of this system should be less than the complexity of the systems considered in Fig. 2, since the system using the 4-antenna space–time code does not need to perform interference cancellation and the decoding is no more complex than that for the systems in Fig. 2. As expected, the 256-state code performs best, as illustrated by the bottom curve in Fig. 3. Again, the improvements increase with increasing Doppler frequency. At 40 Hz Doppler, the system with the 16-state, 4-antenna space–time code is more than 2 dB better than the best system from Fig. 2. Similarly, the system with the 256-state, 4-antenna space–time code is more than 2 dB better than the system with the 16-state, 4-antenna space–time code, at 40 Hz Doppler.

TABLE II
GENERATOR MATRICES FOR THE FOUR TRANSMIT ANTENNA CODES USED IN FIGS. 1 AND 4. THESE CODES ARE IN GF(4) WITH ELEMENTS DENOTED BY $\{0, 1, a, 1+a\}$

$$\begin{pmatrix} (1+a)+D & a+(1+a)D & a+D & 1+(1+a)D \\ a+(1+a)D & a+D & 1+(1+a)D & 1+(1+a)D \\ a+D & 1+(1+a)D & 1+(1+a)D & (1+a)+aD \\ 1+(1+a)D & 1+(1+a)D & (1+a)+aD & 1+aD \end{pmatrix} \text{ for the 16-state code.}$$

$$\begin{pmatrix} (1+a)+(1+a)D+aD^2 & (1+a)D+aD^2 & 1+D^2 & 1+D^2 \\ (1+a)D+aD^2 & 1+D^2 & 1+(1+a)D+(1+a)D^2 & (1+a)+aD+(1+a)D^2 \\ 1+D^2 & 1+(1+a)D+(1+a)D^2 & (1+a)+aD+(1+a)D^2 & (1+a)+aD \\ 1+(1+a)D+(1+a)D^2 & (1+a)+aD+(1+a)D^2 & (1+a)+aD & D+D^2 \end{pmatrix}$$

for the 256-state code.

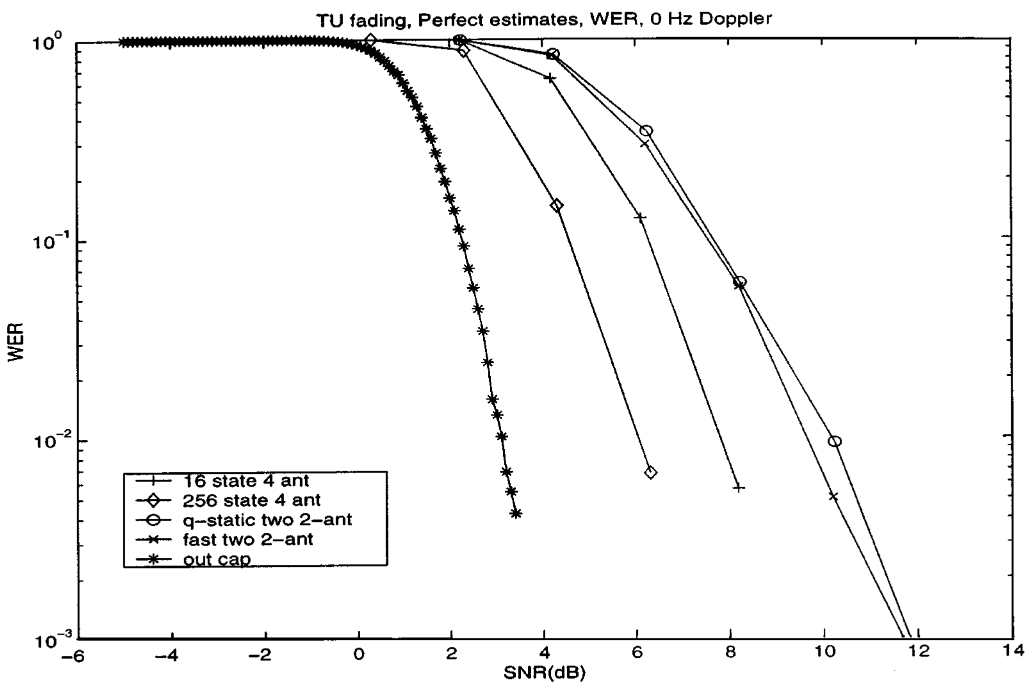


Fig. 4. Comparisons of WER for best MIMO-OFDM systems from Figs. 2 and 3 with perfect estimates and no Doppler. See Tables I and II for details on the codes.

It has become common to compare the WER of a real system with the outage capacity of the capacity-optimized signaling scheme [3], [5]. Here we consider comparing $\mathrm{Prob}(C < 4)$ to WER as per [3], since our system would produce a transmission efficiency of 4 bit/s/Hz ignoring the guard tones and guard intervals. Accounting for these factors will not change the results. Such a comparison is shown in Fig. 4. The performance of the four best MIMO-OFDM systems from Figs. 2 and 3 are shown. In this comparison, perfect estimation and zero Doppler is assumed. Obviously, the outage capacity is given by the lowest curve. The system with the 256-state, 4-antenna space–time code achieves a WER that is about 2 dB from the outage capacity at WER $= 10^{-1}$. Since there was no attempt to optimize the 4-antenna space–time codes, further improvement may be possible, even without complexity increase.

IV. CONCLUSION

Improved MIMO-OFDM techniques were studied for wireless systems using QPSK modulation for four transmit and four receive antennas. We first considered such a system employing two 16-state, 2-antenna space–time codes with successive interference cancellation and channel estimation, which was previously proposed to reduce the complexity of a 4-antenna space–time code system. We showed that our recently proposed space–time code has a 2-dB improvement over a previously published code at 5-Hz fading. Furthermore, we proposed a 4-antenna, 16-state code that achieves an additional 2-dB improvement with *lower* complexity and a 256-state code that achieves an additional 2-dB gain. The 256-state code performed within 3 dB of outage capacity (and within 2 dB with perfect channel estimation).

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