On Optimum MIMO with Antenna Selection

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Abstract— **Wireless communication systems with transmit and receive antenna arrays are studied when antenna selection is used. A case with very limited feedback of information from the receiver to the transmitter is considered, where the only information fed back is the selected subset of transmit antennas to be employed. It is shown that the optimum signaling, for largest ergodic capacity of a single isolated link with given interference and antenna selection, is generally different from that which is optimum without antenna selection for some range of signal-to-noise ratios (SNRs). In cases with interference, the improvement obtained from using the true optimum signaling approach tends to increase for this range of SNRs. Further the optimum approach for cases without antenna selection tends to be optimum in fewer cases as interference power is increased.**

I. Introduction

The great potential for achieving high data rate wireless communications using multiple-input multiple-output (MIMO) channels formed using transmit and receive antenna arrays has been demonstrated [1], [2] and this lure continues to attract attention to this topic. A natural concern in the implementation of such systems is the increased hardware required to implement the multiple RF chains used in a standard multiple transmit and receive antenna array MIMO system. A promising approach for reducing complexity while retaining a reasonably large fraction of the high potential data rate of a MIMO approach appears to be to employ some form of antenna selection [3], [4]. Thus one can employ a reduced number of RF chains at the receiver and attempt to optimally allocate each chain to one of a larger number of receive antennas. In this case only the best set of antennas is used, while the remaining antennas are not employed, thus reducing the number of required RF chains. For cases with only a single transmit antenna where standard diversity reception is to be employed, this approach, known as "hybrid selection/maximum ratio combining", has been shown to lead to relatively small reductions in performance, as compared with using all receive antennas, for considerable complexity reduction [3], [4]. Clearly antenna selection can be simultaneously employed at the transmitter and at the receiver in a MIMO system leading to larger reductions in complexity.

This material is based on research supported by the Air Force Research Laboratory under agreement No. F49620-01-1-0372 and by the National Science Foundation under Grant No. CCR-0112501.

R. Blum was on a sabbatical leave from the EECS Dept., Lehigh University, Bethlehem, PA.

Employing antenna selection both at the transmitter and the receiver in a MIMO system has been studied very recently [5], [6], [7]. Cases with full and limited feedback of information from the receiver to the transmitter have been considered. The cases with limited feedback are especially attractive in that they allow antenna selection at the transmitter without requiring a full description of the channel or its eigenvector decomposition to be fed back. In particular, the only information fed back is the selected subset of transmit antennas to be employed. While cases with this limited feedback of information from the receiver to the transmitter have been studied in these papers, they each assume the transmitter sends a different equal power signal out of each selected antenna. Transmitting a different equal power signal out of each antenna is the optimum approach for the case where selection is not employed [8]. The purpose of this paper is to demonstrate that this approach is not necessarily best in cases where antenna selection is employed, which is a fact that appears not to have been recognized previously. However, we show that this approach can be best in some cases with sufficiently high SNR. For simplicity, we ignore any delay or error that might actually be present in the feedback signal. We assume the feedback signal is accurate and instantly follows any changes in the environment.

II. Model of MIMO channel

First consider an isolated MIMO link with Rayleigh fading and additive white Gaussian noise only (no interference). To simplify matters assume quasi-static flat fading and initially assume antenna selection is not employed. The vector of complex baseband samples from the set of n_r receive antennas after matched filtering is

$$\mathbf{y} = (y_1, \ldots, y_{n_r})^T = \mathbf{H}\mathbf{x} + \mathbf{n} \qquad (1)$$

where $\mathbf{x} = (x_1, \ldots, x_{n_t})^T$ is the transmitted vector, $\mathbf{H}$ is the channel matrix with independent entries that are each zero-mean complex Gaussian fading coefficients and $\mathbf{n} = (n_1, \ldots, n_{n_r})^T$ is the additive zero-mean complex white Gaussian noise vector. For simplicity we assume

n_t, the number of transmit antennas, satisfies $n_t \leq n_r$ although more general cases are easy to handle [8]. If $\mathbf{H}$ is unknown at the transmitter, it is known [8], [2] that the optimum signaling (to maximize ergodic capacity, mutual information between transmitted and received signals) is Gaussian with covariance matrix $\mathbf{Q} = \frac{\rho}{n_t}\mathbf{I}_{n_t}$ where $\mathbf{I}_{n_t}$ is an $n_t \times n_t$ identity matrix and ρ is the fixed total transmit power. Let $\lambda_1 \leq \lambda_2 \cdots \leq \lambda_{n_t}$ be the eigenvalues of $\mathbf{H}\mathbf{H}^H$. Then the mutual infomation conditioned on $\mathbf{H}$ (which we call capacity for shorthand) obtained using this approach is

$$C_M = \log_2 \left(det\left(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{H}\mathbf{H}^H \right)\right) = \sum_{i=1}^{n_t} \log_2 (1 + \frac{\rho}{n_t}\lambda_i). \quad (2)$$

The subscript on C reminds us a MIMO approach is used. Since noise power is normalized, ρ is also the SNR [2].

Now consider a different signaling approach, the single stream signaling introduced in [9]. Let s denote a complex constellation symbol representing elements from the data stream to be transmitted and assume a unit-length transmit weight vector $\mathbf{w}_{t,n_t}$ will be chosen so that $\mathbf{w}_{t,n_t} s$ is transmitted. We pick a fixed transmit weight vector $\mathbf{w}_{t,n_t} = (1/\sqrt{n_t}, \ldots, 1/\sqrt{n_t})^T$. In this case we obtain

$$C_1 = \log_2 \left(1 + \rho|\mathbf{H}\mathbf{w}_{t,n_t}|^2\right). \quad (3)$$

The subscript on C reminds us a single stream approach is used. In [9] we show that $E\{C_1\} \leq E\{\log_2 (1 + \frac{\rho}{n_t}\lambda_{n_t})\} \leq E\{C_M\}$ for cases without antenna selection.

III. Antenna Selection

Now assume that we select $n_{st} < n_t$ transmit antennas and $n_{sr} < n_r$ receive antennas using an antenna selection algorithm. Then the observations from the selected antennas follow the model in (1) with n_t and n_r replaced by n_{st} and n_{sr} respectively and $\mathbf{H}$ replaced by $\tilde{\mathbf{H}}$. $\tilde{\mathbf{H}}$ is obtained by eliminating those columns and rows of $\mathbf{H}$ corresponding to unselected transmit and receive antennas. Thus we can write $\tilde{\mathbf{H}} = f(\mathbf{H})$ where the function f specifies the selection criterion. This criterion might attempt to choose $\tilde{\mathbf{H}}$ to maximize the capacity when a fixed transmission approach is employed, for example zero-mean Gaussian signaling with covariance matrix $\mathbf{Q} = \frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$ or the single stream transmission approach we have outlined. Since the model with antenna selection conditioned on $\tilde{\mathbf{H}}$ is the same as the model without antenna selection conditioned on $\mathbf{H}$ then it follows that with antenna selection

$$C_M = \log_2 \left(det\left(\mathbf{I}_{n_{sr}} + \frac{\rho}{n_{st}}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H \right)\right). \quad (4)$$

where we choose $\tilde{\mathbf{H}} = f(\mathbf{H})$ to maximize C_M and

$$C_1 = \log_2 \left(1 + \rho|\tilde{\mathbf{H}}\mathbf{w}_{t,n_{st}}|^2\right) \quad (5)$$

where we choose $\tilde{\mathbf{H}} = f(\mathbf{H})$ to maximize C_1. Furthermore, it follows from [8] that the optimum signaling is still Gaussian with a covariance matrix $\mathbf{Q}$. However, the optimum $\mathbf{Q}$ is not necessarily $\frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$ as we now show. For simplicity we focus on the case of sufficiently weak signals so that Taylor series approximations are accurate to obtain

$$C_M \approx \frac{\rho}{n_{st}\ln(2)} \left(\sum_{i}^{n_{sr}} \sum_{j=1}^{n_{st}} |\tilde{H}_{ij}|^2 \right). \quad (6)$$

and

$$C_1 \approx \frac{\rho}{n_{st}\ln(2)} \left(\sum_{i=1}^{n_{sr}} \left| \sum_{j=1}^{n_{st}} \tilde{H}_{ij} \right|^2 \right). \quad (7)$$

Furthermore we focus on ergodic capacity (conditional capacity averaged over the random channel). Then the following theorem states that the optimum $\mathbf{Q}$ is not necessarily $\frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$.

Theorem 1

For sufficiently small ρ, $E\{C_1\} > E\{C_M\}$.

Outline of the Proof

The difference between (6) and (7) is the cross terms that appear in (7) which are missing from (6). Specifically, for a given i these are

$$\sum_{j=1}^{n_{st}} \sum_{j'=1, j'\neq j}^{n_{st}} \tilde{H}_{ij}\tilde{H}^*_{ij'} + \tilde{H}_{ij'}\tilde{H}^*_{ij} = \sum_{j=1}^{n_{st}} \sum_{j'=1, j'\neq j}^{n_{st}} 2Re\{\tilde{H}_{ij}\tilde{H}^*_{ij'}\}. \quad (8)$$

In fact there are such cross terms for all valid i. If we always select the same set of transmit and receive antennas then the expected value of each of these cross terms is zero. Now choose two distinct sets of antennas where each set includes transmit and receive antennas. Let $\tilde{\mathbf{H}}'$ denote the matrix $\tilde{\mathbf{H}}$ for the case when we always select the first set of transmit and receive antennas. Let $\tilde{\mathbf{H}}''$ denote the matrix $\tilde{\mathbf{H}}$ for the case where we always choose the second set of antennas. Note that in each of these two cases, antenna selection is not really employed since the same set of antennas is always used. Now consider a selection scheme which employs $\tilde{\mathbf{H}}'$ in all cases except where

$$\sum_{i=1}^{n_{sr}}\sum_{j=1}^{n_{st}}|\tilde{H}''_{ij}|^2 = \sum_{i=1}^{n_{sr}}\sum_{j=1}^{n_{st}}|\tilde{H}'_{ij}|^2,$$

$$\sum_{j=1}^{n_{st}}\sum_{i=1}^{nsr}\sum_{j'=1,j'\neq j}^{n_{st}} 2Re\{\tilde{H}''_{ij}\tilde{H}''^{*}_{ij'}\} > 0$$

and

$$\sum_{j=1}^{n_{st}}\sum_{i=1}^{nsr}\sum_{j'=1,j'\neq j}^{n_{st}} 2Re\{\tilde{H}'_{ij}\tilde{H}'^{*}_{ij'}\} < 0.$$

Our new selection approach will give $C_1 > C_M$ in the exception cases just described. It is key that C_M has no cross terms, so it can't be improved in this way. Since the probability of the exception event is greater than zero under our assumed model, then this antenna selection approach will lead to $E\{C_1\} > E\{C_M\}$. Using these ideas we can always take an optimum selection approach for $E\{C_M\}$ and modify it so $E\{C_1\} > E\{C_M\}$. □

One might wonder why the result in Theorem 1 differs from those in [8]. A very short explanation is that the effective statistics of $\tilde{H}$ are no longer complex Gaussian after selection, which is a needed condition for some of the results in [8]. Numerical results also indicate there are cases where $\mathbf{Q} = \frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$ does not lead to best performance. The ergodic capacity (mutual information) with general $\mathbf{Q}$, is

$$E\{C\} = \Psi(\mathbf{Q},\mathbf{H}) = E\{\log_2\left(det\left(\mathbf{I}_{n_r} + \mathbf{HQH}^H\right)\right)\}. \quad (9)$$

The single stream MIMO approach corresponds to using a constant matrix $\mathbf{Q}$. Using a $\mathbf{Q}$ of this form can sometimes provide better performance than can be obtained using $\mathbf{Q} = \frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$. This is illustrated by the results in Figure 1 for a case with $\rho = -9dB$, $n_t = n_r = 8$, $n_{st} = n_{sr} = 2$. Figure 1 shows a plot of $\Psi(\mathbf{Q},\tilde{\mathbf{H}})$ versus the scalars b and a when

$$\mathbf{Q} = \begin{pmatrix} b & a \\ a & \rho - b \end{pmatrix}. \quad (10)$$

In interpreting Figure 1 we recall that power is fixed so $0 \le b \le \rho$ but due to symmetry only $0 \le b \le \rho/2$ need be considered. Furthermore, due to the definition of a we find $a \le \sqrt{b}\sqrt{\rho - b} \le \rho/2$ which means only points on the left hand side of the curve $a \le \sqrt{b}\sqrt{\rho - b}$ are valid. Thus Figure 1 indicates best performance is achieved with $b = a$ and for the largest possible values of the single scalar $b = a$. Thus the best performance is obtained by using $b = \rho/2$ and $a = \rho/2$. This is further clarified by Figure 2 which shows a plot of $\Psi(\mathbf{Q},\tilde{\mathbf{H}})$ versus the scalar b along the contour with $a = \sqrt{b}\sqrt{\rho - b}$. In fact, the results are drastically different if we consider larger ρ as in Figure 3 which is for $\rho = 0dB$. Here we see $b = \rho/2$ and $a = 0$ give best performance. Thus here the optimum signaling without antenna selection $\mathbf{Q} = \frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$ gives best performance. Similar results are obtained for larger ρ where again the optimum signaling without antenna selection is best. The results in Figure 3 are very similar to the results for the same case (no antenna selection, $\rho = 0dB$, $n_t = n_r = 2$) without antenna selection as illustrated in Figure 4.

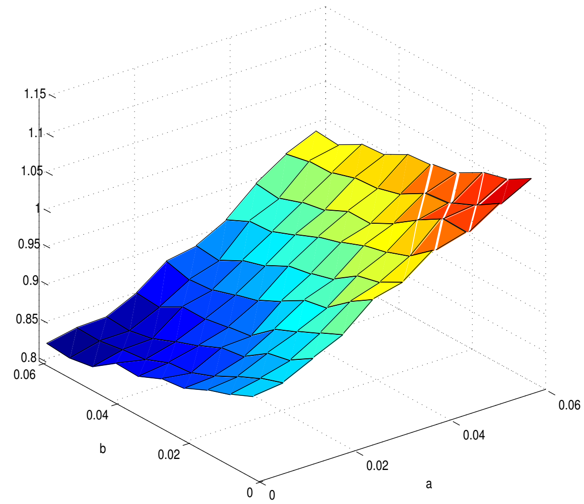

Fig. 1. Ergodic capacity $E\{C\}$ versus b and a with selection and $-9dB$ SNR.

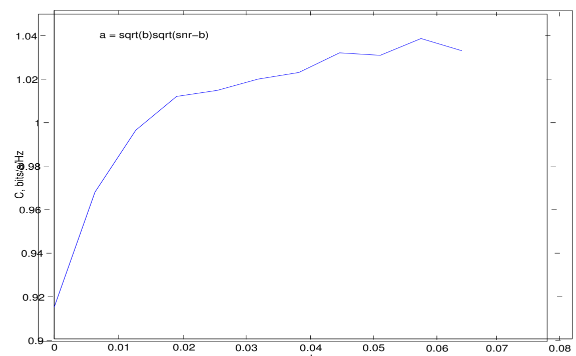

Fig. 2. Ergodic capacity $E\{C\}$ versus b for $a = \sqrt{b}\sqrt{\rho - b}$ with selection and $-9dB$ SNR.

IV. Performance of Signaling Schemes

Next we further study the performance of various MIMO signaling schemes for cases with antenna selection. We generalize consideration to cases with L MIMO

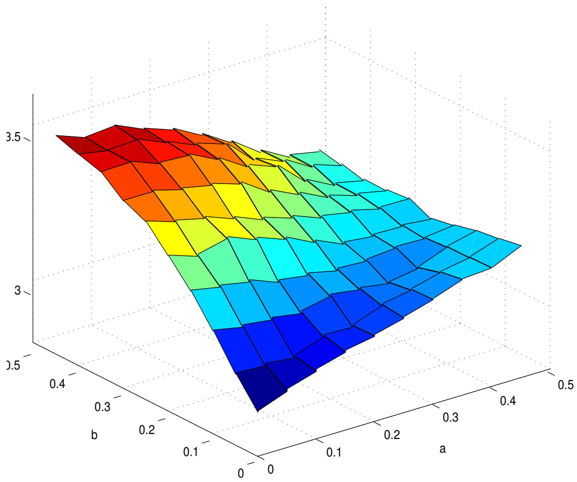

Fig. 3. Ergodic capacity $E\{C\}$ versus b and a with selection and $0dB$ SNR.

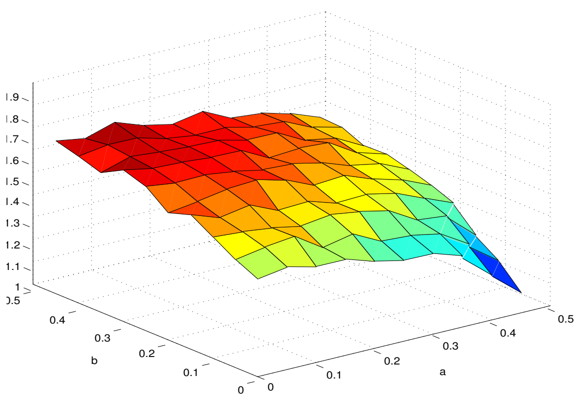

Fig. 4. Ergodic capacity $E\{C\}$ versus b and a without selection and $0dB$ SNR.

interferers so (1) becomes

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \sum_{j=1}^{L} \mathbf{H}_j \mathbf{x}_j + \mathbf{n} \quad (11)$$

where $\mathbf{H}_j$ and $\mathbf{x}_j$ represent the channel matrix and the transmitted signal of user j, respectively. For simplicity, we assume the detailed structure of each of the interfering signals $\mathbf{x}_j, j = 1, \ldots, L$ is unknown to the receiver and we model each of them as Gaussian distributed. Then, if we condition on $\mathbf{H}, \mathbf{H}_1, \ldots, \mathbf{H}_L$, the interference-plus-noise from (11), $\sum_{j=1}^{L} \mathbf{H}_j \mathbf{x}_j + \mathbf{n}$, is Gaussian distributed with the covariance matrix $\mathbf{R} = \sum_{j=1}^{L} \mathbf{H}_j Cov(\mathbf{x}_j) \mathbf{H}_j^H + Cov(\mathbf{n})$ where $Cov(\mathbf{x}_j)$ denotes the covariance matrix of $\mathbf{x}_j$. The interference-plus-noise is whitened by multiplying $\mathbf{y}$ by $\mathbf{R}^{-1/2}$ after which the new version of (11) can be represented using (1) if we take $\hat{\mathbf{H}} = \mathbf{R}^{-1/2}\mathbf{H}$ as the $\mathbf{H}$ which appears in (1), so the optimum signaling for $\mathbf{x}$ is Gaussian. In fact, the Gaussian signaling assumption is exactly the assumption made for the distribution of each $\mathbf{x}_j, j = 1, \ldots, L$, which is reassuring.

We consider the signaling approach which is optimum if selection is not employed $\mathbf{Q} = \frac{\rho}{n_{st}}\mathbf{I}_{n_{st}}$ and also the single stream signaling introduced previously. Our focus will be on a case with $n_t = n_r = 8$, $n_{st} = n_{sr} = 2$. Figure 5 and Figure 6 show the performance of the two approaches, when antenna selection is employed and when it is also not employed (curves labeled fix). Cases are considered where either one ($\mathbf{w}_{\mathbf{t},\mathbf{n_{st}}} = (1/\sqrt{2}, 1/\sqrt{2})^T$, called 1 str, 2 ant) or two streams (called 2 str, 2 ant) are transmitted. The interference comes from a single interferer using either one of two streams with the same interference power in either case. Figure 5 considers the case without interference, while Figure 6 assumes an interference-to-noise ratio (INR) of $10dB$. In both Figure 5 and Figure 6, the improvement due to using selection is clearly seen. This improvement is about $7dB$ (near $E\{C\} = 4$ for example) for the case without interference in Figure 5 (for either one or two streams transmitted). For the case with interference in Figure 6, the improvement due to using selection is slightly larger. Consistent with the analysis in Section II, in Figure 5 we can see that transmitting one stream gives larger ergodic capacity than transmitting two for SNRs below $-2dB$ when antenna selection is employed. With interference, the range of SNR where transmitting one stream gives larger ergodic capacity than transmitting two becomes even larger as might be expected from [9]. In Figure 6 we can see that transmitting one stream gives larger ergodic capacity than transmitting two for SNRs below $7dB$ when antenna selection is employed with $10dB$ INR.

The gain obtained from increasing n_{st} and n_{sr}, the number of antennas used after the selection, is also of great interest. If we use the signaling approach which is best without antenna selection, we get the gains shown in Figure 7. The solid curves show ergodic capacity without interference. The dotted curves show the ergodic capacity for a single stream interferer which is slightly less by a factor which varies with the number of antennas used in the selection. Initially this factor tends to increase as the number of antennas used in the selection is increased, but eventually this factor decreases as the number of antennas used in the selection is increased further. Similar comparisons with various numbers of streams and various amounts and types of interference are more complicated

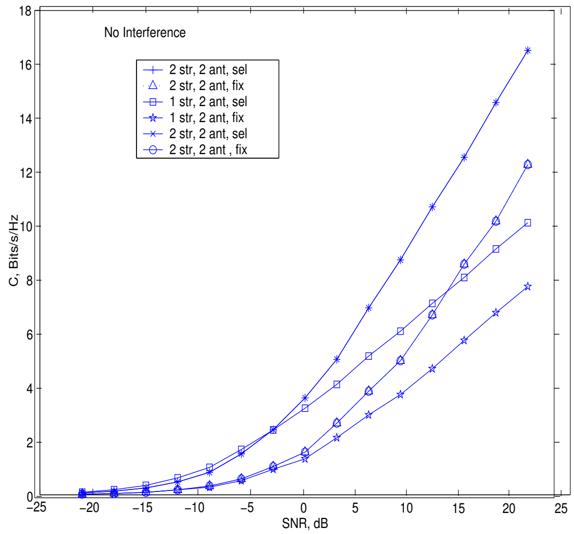

Fig. 5. Ergodic capacity $E\{C\}$ for various approaches with no interference.

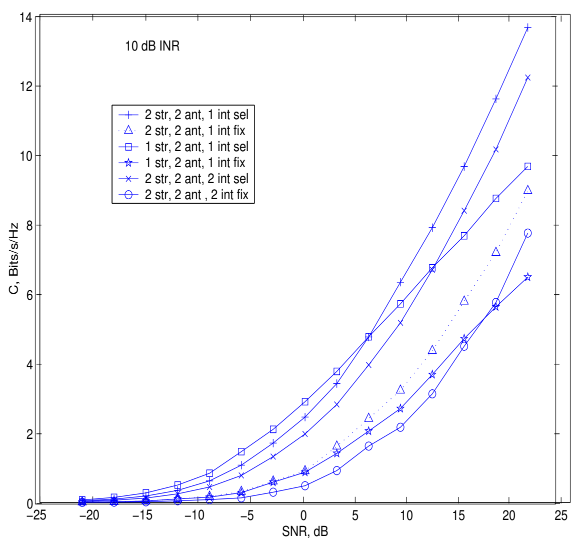

Fig. 6. Ergodic capacity $E\{C\}$ for various approaches with $10dB$ INR.

as one might imagine from [9] and these will be discussed in future papers.

V. Conclusions

The ergodic capacity of MIMO with antenna selection has been studied for cases with limited feedback from the receiver to the transmitter. In particular, the optimum signaling scheme has been considered. It was shown that the optimum signaling for a single, isolated MIMO link, with given interference and antenna selection, is generally different from that which is optimum without antenna selection for some range of signal-to-noise ratios (SNRs). In cases with interference the improvement obtained from using the true optimum signaling approach tends to increase for this range of SNRs. Furthermore the optimum approach for cases without antenna selection tends to be optimum for fewer cases as the interference is increased.

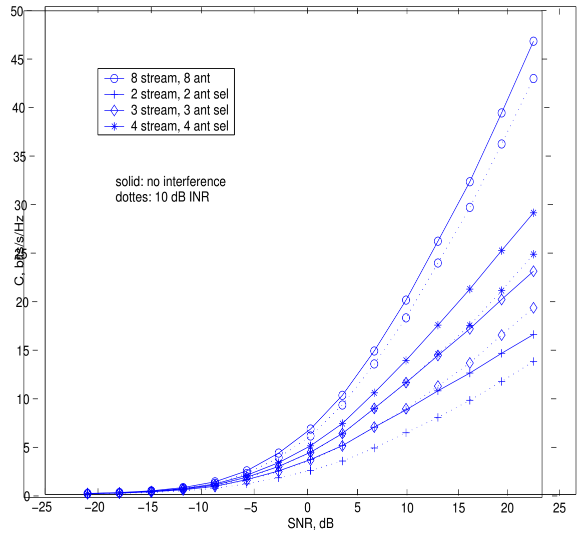

Fig. 7. Ergodic capacity $E\{C\}$ for $n_t = n_r = 8$, and various number of antennas $n_{st} = n_{sr}$ in the selection with no interference (solid) and with $10dB$ INR (dotted).

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