On the Capacity of Cellular Systems With MIMO

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***Abstract*—It is shown that the mutual information of a single, isolated, multiple transmit and receive antenna array link is maximized by transmitting the maximum number of independent data streams for a flat Rayleigh fading channel with independent fading coefficients for each path. However, if such links mutually interfere, in some cases the overall system mutual information can be increased by transmitting fewer streams.**

***Index Terms*—Antenna arrays, channel capacity, MIMO.**

I. Introduction

TRANSMIT and receive antenna arrays used to form multiple-input multiple-output (MIMO) channels have shown great potential in isolated, single link communications without cochannel interference [1], [2]. For a flat Rayleigh fading channel with independent fading, maximum average capacity [largest average mutual information (MI)] is achieved [1], [2] by sending an independent information stream from each transmit antenna which is the maximum possible number of streams which can be sent. Very recent investigations have shown that cochannel interference can seriously degrade the overall capacity [3] when MIMO channels are used in a cellular system. Here we ask if it is always best to send the maximum possible number of independent information streams in order to achieve maximum MI. In particular, we investigate the idea of adaptive MIMO, where the number of independent streams transmitted may be fewer than the maximum possible. Note, the number of streams is less than or equal to the number of transmit antennas. For example, we consider a single stream transmitted by multiple antennas in some cases.

Consider a flat Rayleigh fading channel with independent fading coefficients for each path. First we consider the MI of a single, isolated link with cochannel interference. By isolated we mean that we have no control over the signaling used by the users generating the interference. In particular we cannot control the number of streams they employ. Further, we consider only the MI of the isolated link and do not consider any MI associated with the links which interfere with it. There are some cases of this type where it might first appear to be possible to achieve higher MI by reducing the number of MIMO streams. For example, consider a case with an isolated link using n_t transmit antennas and n_r receive antennas. Assume a single MIMO interferer with k streams. If n_t streams are transmitted, array processing theory implies that the interference can be nulled and reception of the data streams enhanced if $n_t + k \leq n_r$. This suggests that if $n_t + k = n_r + 1$ then the user of interest might achieve higher MI by reducing the number of streams transmitted by at least one. We first show that this is not the case based on an analytical proof. For simplicity, we focus on systems with two transmit antennas. Our analytical proof shows the MI of a single, isolated link is not improved by reducing the number of streams transmitted from two to one. For generality, we actually demonstrate that it is always better to send the maximum number of streams possible as opposed to one stream for cases with any n_t. A numerical example illustrates our point.

On the other hand, if one user in a cellular system uses fewer MIMO streams, this will create fewer cochannel interferers for other users. In fact we show that the MI of each user can be increased if the number of streams transmitted by each user is decreased, for certain signal-to-noise ratios (SNRs) and interference-to-noise ratios (INRs). We demonstrate this by showing that the MI of a two stream user faced with a two stream interferer can be lower than the MI of a one stream user faced with a one stream interferer at certain SNRs and INRs.

II. Model of MIMO Channel

Consider a system where cochannel interference is present from $L-1$ other users. Let us focus on the Lth user and assume each user employs n_t transmit antennas and n_r receive antennas. In this case the vector of received complex baseband samples after matched filtering becomes

$$\mathbf{y}_L = \sqrt{\rho_L}\mathbf{H}_{L,L}\mathbf{x}_L + \sum_{j=1}^{L-1} \sqrt{\eta_{L,j}}\mathbf{H}_{L,j}\mathbf{x}_j + \mathbf{n} \quad (1)$$

where $\mathbf{H}_{L,j}$ and $\mathbf{x}_j$ represent the normalized channel matrix and the normalized transmitted signal of user j respectively. The SNR of user L is ρ_L and the INR for user L due to interference from user j is $\eta_{L,j}$. For simplicity, we assume all of the interfering signals $\mathbf{x}_j, j = 1, \ldots, L-1$ are unknown to the receiver and we model each of them as being Gaussian distributed. Then if we condition on $\mathcal{H}_L = \mathbf{H}_{L,1}, \ldots, \mathbf{H}_{L,L}$, the interference-plus-noise from (1), $\sum_{j=1}^{L-1} \sqrt{\eta_{L,j}}\mathbf{H}_{L,j}\mathbf{x}_j + \mathbf{n}$, is Gaussian distributed with the covariance matrix $\mathbf{R}_L = \sum_{j=1}^{L-1} \eta_{L,j}\mathbf{H}_{L,j}\mathbf{S}_j\mathbf{H}_{L,j}^H + \mathbf{I}_{n_r}$ where $\mathbf{S}_j$ denotes the covariance matrix of $\mathbf{x}_j$ and $\mathbf{I}_{n_r}$ (an $n_r \times n_r$ identity matrix) is the covariance matrix of $\mathbf{n}$. Under this conditioning, the interference-plus-noise is whitened by

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multiplying $\mathbf{y}_L$ by $\mathbf{R}_L^{-1/2}$. After performing this multiplication we can use results from [1], [2] to express the MI between the input and output for the user of interest as in

$$I(\mathbf{x}_L; (\mathbf{y}_L, \mathcal{H}_L)) = E\left\{\log_2\left(\det\left(\mathbf{I}_{n_r} + \rho_L \mathbf{H}_{L,L}\mathbf{S}_L\mathbf{H}_{L,L}^H\mathbf{R}_L^{-1}\right)\right)\right\}. \quad (2)$$

In (2) the identity $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$ was used and we have assumed the receiver knows the realization of $\mathcal{H}_L$.

Since $\mathbf{S}_L$ is positive definite $\mathbf{S}_L = \mathbf{U}_L\mathbf{\Lambda}_L\mathbf{U}_L^H$ where $\mathbf{\Lambda}_L$ is the diagonal matrix of the nonzero eigenvalues of $\mathbf{S}_L$ and $\mathbf{U}_L$ is the unitary matrix with columns consisting of the eigenvectors of $\mathbf{S}_L$. Using $\mathbf{S}_L = \mathbf{U}_L\mathbf{\Lambda}_L\mathbf{U}_L^H$ in (2) and defining $\hat{\mathbf{H}}_{L,L} = \mathbf{H}_{L,L}\mathbf{U}_L$ we obtain an equation in exactly the same form as (2) but with $\mathbf{H}_{L,L} \rightarrow \hat{\mathbf{H}}_{L,L}$ and $\mathbf{S}_L \rightarrow \mathbf{\Lambda}_L$. Since the distribution of $\hat{\mathbf{H}}_{L,L}$ and $\mathbf{H}_{L,L}$ are identical [2] we find the value of $I(\mathbf{x}_L; (\mathbf{y}_L, \mathcal{H}_L))$ from (2) is unchanged when we replace $\mathbf{S}_L$ with $\mathbf{\Lambda}_L$. This tells us that we only need to consider diagonal $\mathbf{S}_L$ to optimize $I(\mathbf{x}_L; (\mathbf{y}_L, \mathcal{H}_L))$ from (2). We call the number of nonzero entries of the diagonal matrix $\mathbf{\Lambda}_L$ the number of streams we employed. We see now we get the same performance (in terms of (2)) if we radiate using $n_s < n_t$ antennas to produce the covariance matrix $\mathbf{\Lambda}_L$ as we do when we radiate using all the antennas using $\mathbf{U}_L\mathbf{\Lambda}_L\mathbf{U}_L^H$. We should mention that in terms of outage capacity the second approach will generally have advantages if properly employed [4]. It is known [2] that the channel will only support $\min(n_t, n_r)$ streams so we call this the maximum number of possible streams and restrict attention to this number of streams or less and assume $n_t \leq n_r$????? for simplicity. Further we use equal power for each stream due to the lack of knowledge of $\mathcal{H}_{\mathcal{L}}$ at the transmitter.

III. Isolated Links and Systems

To compute the capacity of an isolated link we want to maximize (2) over $\mathbf{S}_L$ for given $\mathbf{S}_1, \ldots, \mathbf{S}_{L-1}$. For cases without interference [2] $\mathbf{S}_L = (1/n_t)\mathbf{I}_{n_t}$ is optimum. Thus an independent data stream should be sent from each antenna so that the maximum number of streams possible $n_m = \min(n_t, n_r) = n_t$ should be used. From (2), employing $\mathbf{S}_L = (1/n_t)\mathbf{I}_{n_t}$ in a case with interference gives

$$I(\mathbf{x}_L; (\mathbf{y}_L, \mathcal{H}_L)) = E\left\{\sum_{i=1}^{n_m}\log_2\left(1 + \frac{\rho_L}{n_t}\lambda_i\right)\right\} \quad (3)$$

where $\lambda_1 \leq \lambda_2 \cdots \leq \lambda_{n_m}$ are the eigenvalues of $(\mathbf{R}_L^{-1/2}\mathbf{H}_{L,L})(\mathbf{R}_L^{-1/2}\mathbf{H}_{L,L})^H$. Now we want to demonstrate that it is always best to send as many streams as possible to maximize the average MI of an isolated link, for a given transmit power. To illustrate this point in a simple way, we investigate sending one stream as opposed to sending more streams.

Consider transmitting only one data stream from the transmit antenna array and using optimum linear combining at the receiver. Let s denote a complex constellation symbol representing elements from the data stream to be transmitted and assume a unit-length transmit weight vector $\mathbf{w_t}$ will be chosen so that $\mathbf{w_t}s$ is transmitted. The optimum unit-length combining

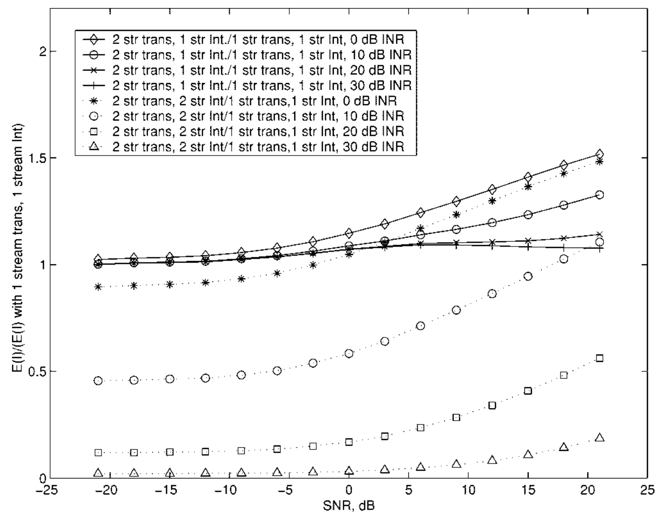

Fig. 1. Solid curves are average mutual information $E\{I\}$ for two transmitted streams divided by the average mutual information for one stream transmitted with the same one stream interference. The dashed curves are average mutual information for one transmitted stream and one interference stream divided by the average mutual information for two transmitted streams and two interference streams. All assume flat fading with either 0-, 10-, 20-, or 30–dB INR.

weight vector $\mathbf{w_r}$ is used at the receiver so the MI obtained using the single stream (and feedback) is (by Cauchy-Schwartz inequality)

$$\begin{aligned}
E&\left\{\max_{\mathbf{w_r}}\log_2\left(1 + \left|\mathbf{w_r}^H\mathbf{R}_L^{-1/2}\mathbf{H}_{L,L}\mathbf{w_t}s\right|^2\right)\right\} \\
&= E\left\{\log_2\left(1 + \rho_L\left|\mathbf{R}_L^{-1/2}\mathbf{H}_{L,L}\mathbf{w_t}\right|^2\right)\right\} \\
&= E\left\{\log_2\left(1 + \hat{\rho}_L\lambda_{n_m}\right)\right\} \\
&\leq E\left\{\log_2\left(1 + E\left\{\hat{\rho}_L\,\lambda_{n_m} \mid \lambda_{n_m}\right\}\right)\right\} \\
&= E\left\{\log_2\left(1 + \frac{\rho_L}{n_t}\lambda_{n_m}\right)\right\} \qquad (4)
\end{aligned}$$

where $\hat{\rho}_L$ is the effective SNR. Thus by a well-known theorem of linear algebra $\hat{\rho}_L$ would equal ρ_L if we could choose $\mathbf{w_t}$ in an optimum way based on exact knowledge of $\mathbf{R}_L^{-1/2}\mathbf{H}_{L,L}$ (requires feedback). In cases without feedback $\hat{\rho}_L \leq \rho_L$ resulting in a loss of capacity. The inequality in (4) is justified by the convexity of $-\log x$ and Jensen's inequality. The final equality in (4) expresses the fact that lack of knowledge of $\mathbf{R}_L^{-1/2}\mathbf{H}_{L,L}$ can be quantified as a loss of the average received SNR (much like a loss of coherence) when compared to the best possible case.

By comparing (3) with (4), it is easy to see that (3) will always be as large as or larger than (4) since (4) is exactly equal to the last term in (3) and the other terms in (3) are nonnegative. Thus it always makes sense to transmit the maximum number of streams when considering an isolated link. The result in (4) can be generalized for cases with more than one stream. These conclusions can also be verified numerically. Here we focus on a case with $n_r = n_t = 2$. Numerical results illustrating the superiority of using two streams are provided in the solid curves in Fig. 1 which show the ratio of the two-stream MI to the one-stream MI versus SNR for a common (0, 10, 20, or 30 dB) INR due to a single stream interferer. Since the ratio is always larger than unity, transmitting two streams is always better for the same single stream interferer. From Fig. 1, it is clear that the increased

MI from transmitting two streams gets smaller as INR increases as expected.

If we wish to compute total system capacity we should find $\mathbf{S}_1, \ldots \mathbf{S}_L$ to maximize

$$\sum_{i=1}^{L} I(\mathbf{x}_i; (\mathbf{y}_i, \mathcal{H}_L)) = \sum_{i=1}^{L} E\left\{ \log_2 \left(\det \left(\mathbf{I}_{n_r} + \rho_i \mathbf{H}_{i,i} \mathbf{S}_i \mathbf{H}_{i,i}^H \left(\mathbf{I}_{n_r} + \sum_{j=1, j\neq i}^{L} \eta_{i,j} \mathbf{H}_{i,j} \mathbf{S}_j \mathbf{H}_{i,j}^H \right)^{-1} \right) \right) \right\}. \tag{5}$$

It is easy to see that there is no loss in optimality from considering diagonal $\mathbf{S}_j, j = 1, \ldots, L$ in (5) using similar arguments as used for (2).

IV. MIMO Stream Control

A key issue for system capacity is that interference from one user will hurt another. In fact, it may be better to have all users use fewer than the maximum number of possible streams in order to increase the MI of each user. To illustrate this consider a particular case with $n_t = n_r = L = 2$ where each user experiences interference from one other user who uses the same number of streams as they do. Each user also has the same SNR $(\rho_1 = \rho_2)$ and INR $(\eta_{2,1} = \eta_{1,2})$. Thus we consider the value of (5) achieved by a system that uses $\mathbf{S}_1 = (1/n_t)\mathbf{I}_{n_t}$ and $\mathbf{S}_2 = (1/n_t)\mathbf{I}_{n_t}$ where all users use two streams. We compare this to the value of (5) achieved by a system that uses $\mathbf{S}_1 = \mathbf{S}_2 = \mathrm{diag}(1, 0)$ where all users use a single stream. The ratios of these quantities are provided in the dashed curves in Fig. 1. Here we find that if all users use a single stream, the system MI in (5) is often higher than if all users use the maximum possible number of streams, which is two in this case. Reducing the number of streams transmitted is somewhat similar to power control. Thus other users see more favorable interference environments when you control the number of streams you employ. In particular, using fewer streams frees up dimensions for others to signal within while still experiencing no interference. One might wonder exactly how important it is to select the correct number of streams. Fig. 2 shows the difference in MI using two streams versus one stream desired and interference signals as a function of SNR and INR using a contour plot. If SNR and INR can be estimated, such a plot could be used to select if one or two streams should be used by two users that are mutually interfering. The contour plot shows clearly the loss in MI that would result by not making the selection correctly for a given SNR and INR. In fact, one can imagine more complicated schemes similar to the power control algorithms typically used in cellular systems that would control the number of streams used by each user. One observation is that for SNR and INR between 0 and 10 dB, which holds strong interest in cellular systems, there is a modest difference between the MI of one-stream and two-stream systems.

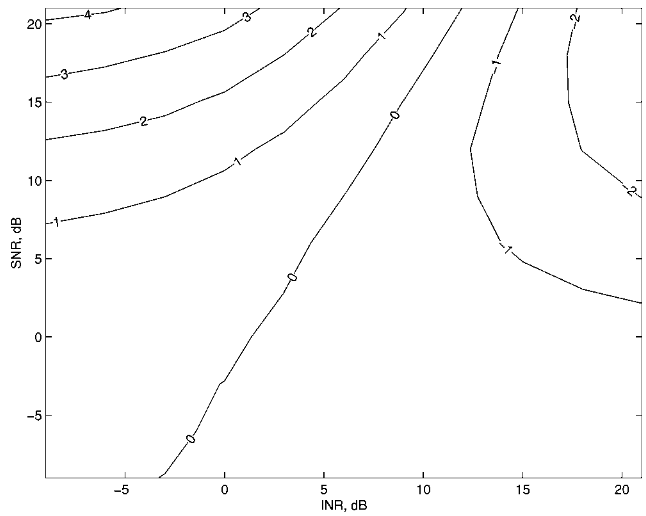

Fig. 2. Average mutual information (two streams for desired and interference signals)—average mutual information (one stream for desired and interference signals) versus SNR and INR.

V. Conclusion

We have analyzed MIMO capacity with interference. We have shown that reducing the number of streams transmitted by all users can provide benefits for system MI. Here we have not shown that the optimum signaling will use $\mathbf{S}_1 = \mathbf{S}_2 = \mathrm{diag}(1, 0)$ for $n_t = n_r = 2$ in some cases. However we have shown this in a companion paper [5]. We have introduced the interesting idea of stream control. Similar results have been obtained [4] for cases with various numbers of antennas, feedback, and outage capacity. It is clear that the results given can be easily extended to OFDM communication systems with nonflat fading [6], and an arbitrary number of cochannel users. We note that the results presented here hold only for infinite size constellations and infinitely complex coding since they are based on capacity. For a discussion of cases without coding, the interested reader is referred to [7].

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