Transactions Letters

Performance of Dual-Diversity Predetection EGC in Correlated Rayleigh Fading With Unequal Branch SNRs

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***Abstract*—The bit error probability (BEP) for coherent detection of binary signals with dual-diversity predetection equal gain combining is derived using the Beaulieu series. In particular, we consider a correlated Rayleigh fading channel with unequal branch signal-to-noise ratios. The BEP expression is in terms of the power correlation coefficient of the branches, is easy to compute, and depicts clearly the effect of correlated fading on the error performance.**

***Index Terms*—Bit error probability, coherent binary keying, correlated Rayleigh fading.**

I. Introduction

AMONG the suboptimal diversity combining methods to combat fading in wireless communications, predetection equal gain combining (EGC) provides a performance comparable to that of maximal ratio combining (MRC), but with a simplified receiver structure. Hence, analysis of EGC is of considerable interest. EGC for independent diversity branches in Rayleigh and Nakagami fading was studied in [1]–[5].

In this letter, we analyze predetection EGC with dual-diversity in correlated Rayleigh fading. Using the Beaulieu series [6], we derive a computationally simple series expression for the average bit error probability (BEP) of coherent detection of binary signals. Both the cases of equal branch signal-to-noise ratios (SNRs) and unequal branch SNRs are included in our framework.

In Section II, we present preliminary results related to the bivariate Rayleigh distribution. An expression for the BEP is obtained in Section III. Section IV gives some numerical results.

II. Preliminaries

Let α_1 and α_2 be two correlated random variables which are marginally Rayleigh with second moments $\mathbb{E}\{\alpha_i^2\} = \Omega_i, i = 1, 2$ where $\mathbb{E}\{\cdot\}$ denotes the expectation, and *power correlation coefficient*

$$\frac{\operatorname{cov}\left(\alpha_1^2, \alpha_2^2\right)}{\sqrt{\operatorname{var}\left(\alpha_1^2\right)\operatorname{var}\left(\alpha_2^2\right)}} = \rho, \quad 0 \le \rho < 1. \tag{1}$$

The joint cumulative distribution function (cdf) of α_1, α_2 can be expressed in terms of the Beaulieu series [6]

$$\begin{aligned} F_{\alpha_1,\alpha_2}(r_1, r_2) &= (1-\rho)\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2}\gamma\left(k+1, \frac{r_1^2}{\Omega_1(1-1\rho)}\right) \\ &\quad \times \gamma\left(k+1, \frac{r_2^2}{\Omega_2(1-\rho)}\right), \qquad r_1, r_2 \ge 0 \end{aligned} \tag{2}$$

where the incomplete gamma function $\gamma(k+1, x)$ has the representation [7, eq. 8.352(1)]

$$\gamma(k+1, x) = \int_0^x u^k e^{-u}\, du = k!\left[1 - e^{-x}\sum_{i=0}^{k}\frac{x^i}{i!}\right].$$

In the performance analysis, we will use the joint characteristic function (cf) of α_1 and α_2, which can be obtained from (2) and is given by [8, p. 409]

$$\begin{aligned} \Psi_{\alpha_1,\alpha_2}(\jmath\omega_1, \jmath\omega_2) &= \mathbb{E}\left\{e^{\jmath(\omega_1\alpha_1+\omega_2\alpha_2)}\right\} \\ &= (1-\rho)\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2}G_k(\jmath\omega_1; \Omega_1, \rho) \\ &\quad \times G_k(\jmath\omega_2; \Omega_2, \rho) \end{aligned} \tag{3}$$

where for $i = 1, 2$

$$\begin{aligned} G_k(\jmath\omega_i; \Omega_i, \rho) &= \Gamma(k+1)\,{}_1F_1\left(k+1; \frac{1}{2}; -\frac{\omega_i^2\Omega_i(1-\rho)}{4}\right) \\ &\quad + \jmath\omega_i[\Omega_i(1-\rho)]^{\frac{1}{2}}\Gamma\left(k+\frac{3}{2}\right) \\ &\quad \times {}_1F_1\left(k+\frac{3}{2}; \frac{3}{2}; -\frac{\omega_i^2\Omega_i(1-\rho)}{4}\right). \end{aligned} \tag{4}$$

Note that $\Gamma(\cdot)$ denotes the gamma function and ${}_1F_1(\cdot;\cdot;\cdot)$ denotes the confluent hypergeometric function [7, eq. 9.210(1)].

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III. Equal Gain Combining

We consider a coherent dual-diversity reception system with a correlated flat Rayleigh fading channel, in which the receiver employs matched filter detection. With EGC, the received signals of each diversity branch are co-phased, combined, and coherently demodulated. The complex baseband signal received over the kth diversity branch in a bit interval $0 \le t < T_b$ can be represented as

$$r_k(t) = \alpha_k e^{-j\theta_k} s(t) + n_k(t), \qquad k = 1, 2$$

where $s(t)$ is the information-bearing signal, α_k and θ_k are the fading magnitude and phase, respectively, of the kth diversity branch, and $n_k(t)$ represents the additive noise. The noises $n_1(t)$ and $n_2(t)$ are assumed to be independent zero-mean complex white Gaussian random processes with two-sided power spectral densities $2N_{01}$ and $2N_{02}$, respectively. We also assume independence among the random sequences $\{\alpha_k\}, \{\theta_k\}$, and $\{n_k(t)\}$. The fading magnitudes α_1, α_2 are assumed to be *correlated* Rayleigh random variables satisfying (1) with joint cdf $F_{\alpha_1,\alpha_2}(\cdot,\cdot)$ given by (2).

Consider coherent detection of binary signals in which, over a bit interval, $s(t) = s_i(t)$ if bit i is transmitted, where $i = 0, 1$. The complex waveforms $s_0(t)$ and $s_1(t)$ have support $[0, T_b)$ and satisfy[1]

$$\int_0^{T_b} |s_i(t)|^2\, dt = 2E_s, \qquad i = 0, 1$$
$$\Re\left\{\int_0^{T_b} s_1(t) s_0^*(t)\, dt\right\} = 2\epsilon E_s, \quad -1 \le \epsilon < 1. \qquad (5)$$

Note that the *signal correlation coefficient* ϵ is the correlation coefficient of $s_0(t)$ and $s_1(t)$. The decision rule of the receiver is given by

$$\Re\left\{\sum_{k=1}^{2} e^{j\theta_k} \int_0^{T_b} r_k(t)\left[s_1^*(t) - s_0^*(t)\right] dt\right\} \overset{1}{\underset{0}{\gtrless}} 0.$$

This can be simplified using (5) to yield [2]

$$D_i \triangleq (-1)^{(i+1)}(\alpha_1 + \alpha_2) + (W_1 + W_2) \overset{1}{\underset{0}{\gtrless}} 0, \quad \text{if bit } i \text{ is transmitted}, \qquad i = 0, 1 \qquad (6)$$

where

$$W_k = \frac{1}{2E_s(1-\epsilon)} \Re\left\{e^{j\theta_k} \int_0^{T_b} n_k(t)\left[s_1^*(t) - s_0^*(t)\right] dt\right\}, \qquad k = 1, 2$$

are independent zero-mean real Gaussian random variables with variances

$$\mathbb{E}\left\{W_k^2\right\} = \frac{N_{0k}}{E_s(1-\epsilon)}, \qquad k = 1, 2.$$

[1]The notation $\Re\{\cdot\}$ stands for the real-part operator.

By the symmetry of $W_1 + W_2$, the BEP is given by $P_{e,\text{EGC}} = \mathbb{P}\{D_1 < 0\}$, where $\mathbb{P}\{\cdot\}$ represents the probability. If $F_{D_1}(\cdot)$ denotes the cdf of the decision variable D_1 and $\Psi_{D_1}(j\omega)$ denotes its cf, then, invoking the inversion theorem [9], we get[2]

$$P_{e,\text{EGC}} = F_{D_1}(0) = \frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\Im\{\Psi_{D_1}(j\omega)\}}{\omega}\, d\omega. \qquad (7)$$

The cf of $D_1 = (\alpha_1 + \alpha_2) + (W_1 + W_2)$ is given by

$$\Psi_{D_1}(j\omega) = \Psi_{W_1}(j\omega)\Psi_{W_2}(j\omega)\Psi_{\alpha_1+\alpha_2}(j\omega)$$
$$= e^{-\frac{\omega^2(N_{01}+N_{02})}{2E_s(1-\epsilon)}} \Psi_{\alpha_1,\alpha_2}(j\omega, j\omega) \qquad (8)$$

where $\Psi_{\alpha_1,\alpha_2}(j\omega, j\omega)$ can be obtained from (3). Noting that

$$\Gamma\left(k + \frac{3}{2}\right) = \frac{(2k+1)!}{k!2^{2k+1}}\pi^{\frac{1}{2}}$$

we get from (8) and (3) the relation

$$\frac{\Im\{\Psi_{D_1}(j\omega)\}}{\omega} = e^{-\frac{\omega^2(N_{01}+N_{02})}{2E_s(1-\epsilon)}} \frac{(1-\rho)^{\frac{3}{2}}}{2\pi^{-\frac{1}{2}}} \sum_{k=0}^{\infty} \frac{(2k+1)!\rho^k}{(k!)^2 4^k}$$
$$\times \left[\Omega_1^{\frac{1}{2}} g_k(\omega; \Omega_1[1-\rho], \Omega_2[1-\rho]) + \Omega_2^{\frac{1}{2}} g_k(\omega; \Omega_2[1-\rho], \Omega_1[1-\rho])\right] \qquad (9a)$$

where

$$g_k(\omega; a_1, a_2) = {}_1F_1\left(k + \frac{3}{2}; \frac{3}{2}; -\frac{\omega^2 a_1}{4}\right) \times {}_1F_1\left(k+1; \frac{1}{2}; -\frac{\omega^2 a_2}{4}\right). \qquad (9b)$$

Now [8, p. 1074]

$${}_1F_1\left(k + \frac{3}{2}; \frac{3}{2}; -x\right) = \frac{(-1)^k 2^{k+1}(k+1)!}{(2k+2)!} \times \frac{e^{-x}}{\sqrt{2x}} He_{2k+1}(\sqrt{2x}) \qquad (10)$$

where $He_{2k+1}(\cdot)$ is the Hermite polynomial [10, eq. 22.3.11] of order $2k+1$.

We next evaluate the integral

$$h_k(a_0, a_1, a_2) \triangleq \int_{-\infty}^{\infty} e^{-\frac{\omega^2 a_0}{4}} g_k(\omega; a_1, a_2)\, d\omega. \qquad (11)$$

Changing the variable of integration to $u = \omega^2$ in (11), and using (9b) and (10), we obtain

$$h_k(a_0, a_1, a_2) = (-1)^k k! \sum_{m=0}^{k} \frac{(-1)^m}{m!(2k+1-2m)!} a_1^{k-m}$$
$$\times \int_0^{\infty} u^{\left(k-m+\frac{1}{2}\right)-1} e^{-\frac{u(a_1+a_0)}{4}}$$
$$\times {}_1F_1\left(k+1; \frac{1}{2}; -\frac{ua_2}{4}\right) du. \qquad (12)$$

[2]The notation $\Im\{\cdot\}$ stands for the imaginary-part operator.

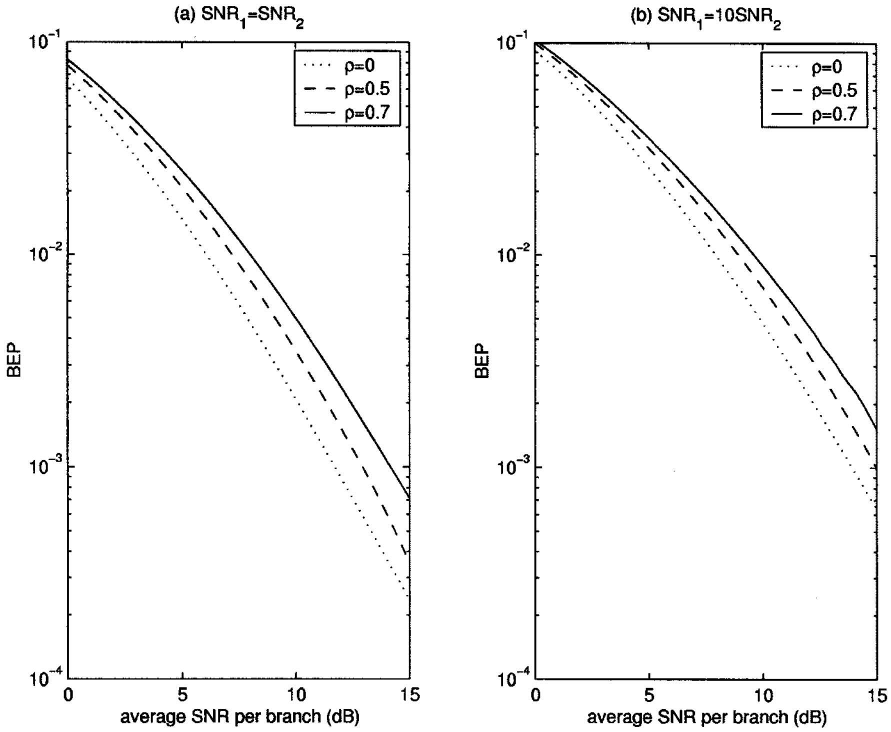

Fig. 1. BEP $P_{e,\mathrm{EGC}}$ of BPSK versus average SNR per branch $(\mathrm{SNR}_1 + \mathrm{SNR}_2)/2$ with different values of power correlation coefficient ρ for (a) $\mathrm{SNR}_1 = \mathrm{SNR}_2$, (b) $\mathrm{SNR}_1 = 10\ \mathrm{SNR}_2$.

Using the result of [7, eq. 7.621(4)] in (12), we get

$$\begin{aligned} h_k(a_0, a_1, a_2) &= (-1)^k k! \sum_{m=0}^{k} \frac{(-1)^m}{m!(2k+1-2m)!} a_1^{k-m} \\ &\times \Gamma\left(k-m+\frac{1}{2}\right)\left(\frac{a_0+a_1+a_2}{4}\right)^{-\left(k-m+\frac{1}{2}\right)} \\ &\times {}_2F_1\left(-k-\frac{1}{2}, k-m+\frac{1}{2}; \frac{1}{2}; \frac{a_2}{a_0+a_1+a_2}\right) \end{aligned} \quad (13)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes the Gaussian hypergeometric function [10, eq. 15.1.1]. Equations (9), (11), and (13) yield

$$\begin{aligned} \int_{-\infty}^{\infty} & \frac{\Im\{\Psi_{D_1}(\jmath\omega)\}}{\omega} d\omega \\ &= \frac{(1-\rho)^{\frac{3}{2}}}{2\pi^{-\frac{1}{2}}} \sum_{k=0}^{\infty} \frac{(2k+1)!\rho^k}{(k!)^2 4^k} \\ &\times \left[\Omega_1^{\frac{1}{2}} h_k\left(\frac{2(N_{01}+N_{02})}{E_s(1-\epsilon)}, \Omega_1[1-\rho], \Omega_2[1-\rho]\right)\right. \\ &\left. + \Omega_2^{\frac{1}{2}} h_k\left(\frac{2(N_{01}+N_{02})}{E_s(1-\epsilon)}, \Omega_2[1-\rho], \Omega_1[1-\rho]\right)\right]. \end{aligned} \quad (14)$$

Let the branch SNRs be defined by [2]

$$\mathrm{SNR}_1 = \frac{\Omega_1 E_s}{\left(\frac{N_{01}+N_{02}}{2}\right)}, \quad \mathrm{SNR}_2 = \frac{\Omega_2 E_s}{\left(\frac{N_{01}+N_{02}}{2}\right)}$$

and let

$$g = \frac{1-\epsilon}{2}.$$

Changing the summation index m to $k-m$ in (13), we combine (14), (13), and (7) to obtain the final expression for the BEP, which is

$$\begin{aligned} P_{e,\mathrm{EGC}} &= \frac{1}{2} - \frac{(1-\rho)}{2} \sum_{k=0}^{\infty} \binom{2k}{k} \frac{\rho^k}{4^k} \sum_{m=0}^{k} \binom{k}{m} (-1)^m \left(\frac{2k+1}{2m+1}\right) \\ &\times \left[\left(\frac{\mathrm{SNR}_1}{\frac{2}{g[1-\rho]} + \mathrm{SNR}_1 + \mathrm{SNR}_2}\right)^{m+\frac{1}{2}}\right. \\ &\times {}_2F_1\left(-k-\frac{1}{2}, m+\frac{1}{2}; \frac{1}{2}; \frac{\mathrm{SNR}_2}{\frac{2}{g[1-\rho]} + \mathrm{SNR}_1 + \mathrm{SNR}_2}\right) \\ &+ \left(\frac{\mathrm{SNR}_2}{\frac{2}{g[1-\rho]} + \mathrm{SNR}_1 + \mathrm{SNR}_2}\right)^{m+\frac{1}{2}} \times {}_2F_1\left(-k-\frac{1}{2}, m\right. \\ &\left.\left. + \frac{1}{2}; \frac{1}{2}; \frac{\mathrm{SNR}_1}{\frac{2}{g[1-\rho]} + \mathrm{SNR}_1 + \mathrm{SNR}_2}\right)\right]. \end{aligned} \quad (15)$$

Expression (15) is in terms of a series of powers of ρ, enabling easy computation of the BEP, and quantifying the effect of ρ on the BEP. In the case of independent branches ($\rho = 0$), only the $k = 0$ term of the summation over k in (15) is nonzero. Using the result [10, eq. 15.1.17]

$${}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; \frac{1}{2}; x\right) = (1-x)^{\frac{1}{2}}$$

(15) simplifies to

$$P_{e,\mathrm{EGC}}\,|_{\rho=0} = \frac{1}{2}\left\{1 - \frac{\sqrt{\mathrm{SNR}_1\left(\mathrm{SNR}_1+\frac{2}{g}\right)}+\sqrt{\mathrm{SNR}_2\left(\mathrm{SNR}_2+\frac{2}{g}\right)}}{\mathrm{SNR}_1+\mathrm{SNR}_2+\frac{2}{g}}\right\}$$

which is the same as [2, eq. (23)].

IV. Numerical Results

The BEP $P_{e,\mathrm{EGC}}$ of BPSK ($g = 1$) is plotted against $(\mathrm{SNR}_1 + \mathrm{SNR}_2)/2$, the average SNR per branch, in Fig. 1, with different values of the power correlation coefficient ρ for equal branch SNRs ($\mathrm{SNR}_1 = \mathrm{SNR}_2$) as well as for unequal branch SNRs ($\mathrm{SNR}_1 = 10\ \mathrm{SNR}_2$). The Gaussian hypergeometric functions in (15) have been calculated using a truncated series formula with a relative error tolerance of 0.001. In computing $P_{e,\mathrm{EGC}}$, 25 terms are taken in the summation over k in (15) for $\rho = 0.5, 0.7$. The maximum relative error obtained over all $P_{e,\mathrm{EGC}}$ computations is 4.04%. The plots reveal that, as expected, for a given average SNR per branch, the BEP increases with an increase in ρ and that a system with equal branch SNRs performs better than that with unequal branch SNRs.

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