On Optimum MIMO With Antenna Selection

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***Abstract*—Wireless communication systems with transmit and receive antenna arrays are studied when antenna selection is used. A case with very limited feedback of information from the receiver to the transmitter is considered, where the only information fed back is the selected subset of transmit antennas to be employed. It is shown that the optimum signaling, for largest ergodic capacity with antenna selection, is generally different from that which is optimum without antenna selection for some range of signal-to-noise ratios.**

***Index Terms*—Antenna arrays, antenna selection, channel capacity, fading channels, MIMO.**

I. INTRODUCTION

THE GREAT potential for achieving high data rate wireless communications using multiple-input multiple-output (MIMO) channels formed using transmit and receive antenna arrays has been demonstrated [1], [2] and this lure continues to attract attention to this topic. A natural concern in the implementation of such systems is the increased hardware required to implement the multiple RF chains used in a standard multiple transmit and receive antenna array MIMO system. A promising approach for reducing complexity while retaining a reasonably large fraction of the high potential data rate of a MIMO approach appears to be to employ some form of antenna selection [3], [4]. Thus one can employ a reduced number of RF chains at the receiver and attempt to optimally allocate each chain to one of a larger number of receive antennas. In this case only the best set of antennas is used, while the remaining antennas are not employed, thus reducing the number of required RF chains. For cases with only a single transmit antenna where standard diversity reception is to be employed, this approach, known as "hybrid selection/maximum ratio combining," has been shown to lead to relatively small reductions in performance, as compared with using all receive antennas, for considerable complexity reduction [3], [4]. Clearly antenna selection can be simultaneously employed at the transmitter and at the receiver in a MIMO system leading to larger reductions in complexity.

Employing antenna selection both at the transmitter and the receiver in a MIMO system has been studied very recently [5]–[7]. Cases with full and limited feedback of information from the receiver to the transmitter have been considered. The cases with limited feedback are especially attractive in that they allow antenna selection at the transmitter without requiring a full description of the channel or its eigenvector decomposition to be fed back. In particular, the only information fed back is the selected subset of transmit antennas to be employed. While cases with this limited feedback of information from the receiver to the transmitter have been studied in these papers, they each assume the transmitter sends a different equal power signal out of each selected antenna. Transmitting a different equal power signal out of each antenna is the optimum approach for the case where selection is not employed [8]. The purpose of this paper is to demonstrate that this approach is not necessarily best in cases where antenna selection is employed, which is a fact that appears not to have been recognized previously. However, we show that this approach can be best in some cases with sufficiently high SNR. For simplicity, we ignore any delay or error that might actually be present in the feedback signal. We assume the feedback signal is accurate and instantly follows any changes in the environment.

II. MODEL OF MIMO CHANNEL

First consider an isolated MIMO link with Rayleigh fading and additive white Gaussian noise only (no interference). To simplify matters assume quasistatic flat fading and initially assume antenna selection is not employed. The vector of complex baseband samples from the set of n_r receive antennas after matched filtering is

$$\mathbf{y} = (y_1, \ldots, y_{n_r})^T = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_{n_t})^T$ is the transmitted vector, $\mathbf{H}$ is the channel matrix with independent entries that are each zero-mean complex Gaussian fading coefficients, and $\mathbf{n} = (n_1, \ldots, n_{n_r})^T$ is the additive zero-mean complex white Gaussian noise vector. For simplicity we assume n_t, the number of transmit antennas, satisfies $n_t \leq n_r$ although more general cases are easy to handle [8]. If $\mathbf{H}$ is unknown at the transmitter, it is known [8], [2] that the optimum signaling (to achieve ergodic capacity, maximum mutual information between transmitted and received signals) is Gaussian with covariance matrix $\mathbf{Q} = (\rho/n_t)\mathbf{I}_{n_t}$ where $\mathbf{I}_{n_t}$ is an $n_t \times n_t$ identity matrix and ρ is the fixed total transmit power. Let $\lambda_1 \leq \lambda_2 \cdots \leq \lambda_{n_t}$ be the eigenvalues of $\mathbf{H}\mathbf{H}^H$. Then the mutual information conditioned on $\mathbf{H}$ obtained using this approach is

$$C_M = \log_2\left(\det\left(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{H}\mathbf{H}^H\right)\right) = \sum_{i=1}^{n_t} \log_2\left(1 + \frac{\rho}{n_t}\lambda_i\right). \tag{2}$$

The subscript on C reminds us a MIMO approach is used. Since noise power is normalized, ρ is also the SNR [2].

Now consider a different signaling approach, the single stream signaling introduced in [9]. Let s denote a complex constellation symbol representing elements from the data stream to be transmitted and assume a unit-length transmit weight vector $\mathbf{w}_{t,n_t}$ will be chosen so that $\mathbf{w}_{t,n_t}s$ is transmitted. For the

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single stream approach we always pick a fixed transmit weight vector $\mathbf{w}_{t,n_t} = (1/\sqrt{n_t}, \ldots, 1/\sqrt{n_t})^T$ in this paper. In this case we obtain (with optimum linear combining)

$$C_1 = \log_2 (1 + \rho |\mathbf{H}\mathbf{w}_{t,\,n_t}|^2). \tag{3}$$

The subscript on C reminds us a single stream approach is used. In [9] we show that $E\{C_1\} \leq E\{\log_2 (1 + (\rho/n_t)\lambda_{n_t})\} \leq E\{C_M\}$ for cases without antenna selection.

III. Antenna Selection

Now assume that we select $n_{st} < n_t$ transmit antennas and $n_{sr} < n_r$ receive antennas using an antenna selection algorithm. Then the observations from the selected antennas follow the model in (1) with n_t and n_r replaced by n_{st} and n_{sr} respectively and $\mathbf{H}$ replaced by $\tilde{\mathbf{H}}$. $\tilde{\mathbf{H}}$ is obtained by eliminating those columns and rows of $\mathbf{H}$ corresponding to unselected transmit and receive antennas. Thus we can write $\tilde{\mathbf{H}} = g(\mathbf{H})$ where the function g specifies the selection criterion. This criterion will choose $\tilde{\mathbf{H}}$ to maximize the capacity when a fixed transmission approach is employed, for example zero-mean Gaussian signaling with covariance matrix $\mathbf{Q} = (\rho/n_{st})\mathbf{I}_{n_{st}}$ or the single stream transmission approach we have outlined.

With selection, it follows from [8] that the optimum signaling is still Gaussian with a covariance matrix $\mathbf{Q}$. However, the optimum $\mathbf{Q}$ is not necessarily $(\rho/n_{st})\mathbf{I}_{n_{st}}$ as we now show. For simplicity we focus on the case of sufficiently weak signals so that Taylor series approximations are accurate to obtain (with selection)

$$C_M \approx \frac{\rho}{n_{st}\ln(2)} \left(\sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} |\tilde{H}_{ij}|^2 \right) \tag{4}$$

$$C_1 \approx \frac{\rho}{n_{st}\ln(2)} \left(\sum_{i=1}^{n_{sr}} \left| \sum_{j=1}^{n_{st}} \tilde{H}_{ij} \right|^2 \right). \tag{5}$$

Furthermore we focus on the mutual information averaged over the random channel. Then the following theorem states that $\mathbf{Q} = (\rho/n_{st})\mathbf{I}_{n_{st}}$ is not the optimum covariance matrix in some cases.

Theorem 1: For sufficiently small ρ, $E\{C_1\} > E\{C_M\}$ when optimum antenna selection is employed for both cases.

Outline of the Proof: First consider the antenna selection approach which maximizes the ergodic mutual information for signaling employing $\mathbf{Q} = (\rho/n_{st})\mathbf{I}_{n_{st}}$. Thus the selection approach will maximize $E\{C_M\}$ by selecting antennas based on the current $\mathbf{H}$ to make (4) as large as possible. It is important to note the choice depends on the squared magnitude of $\tilde{H}_{ij}$. This causes $E\{C_M\} = E\{C_1\}$ if this same selection approach is applied to the single stream signaling in (5) which we will now justify.

The difference between (4) and (5) is the cross terms that appear in (5) which are missing from (4). Specifically, for a given i these are

$$\sum_{j=1}^{n_{st}} \sum_{j'=1,\, j' \neq j}^{n_{st}} \tilde{H}_{ij}\tilde{H}_{ij'}^*. \tag{6}$$

If we use the selection approach that is optimum for $E\{C_M\}$, the cross terms will be averaged to zero due to the symmetry in the selection criterion. First note that the contribution to the ergodic mutual information due to the cross terms in (6) is

$$\int \cdots \int_{H_{11},\ldots,H_{n_t,n_r}} \left(\sum_{j=1}^{n_{st}} \sum_{j'=1,\, j'\neq j}^{n_{st}} \tilde{H}_{ij}\tilde{H}_{ij'}^* \right) f_{H_{11},\ldots,H_{n_t,n_r}}(H_{11},\ldots,H_{n_t,n_r})\, dH_{11}\cdots dH_{n_t,n_r} \tag{7}$$

times the constant $\rho/(n_{st}\ln(2))$. In (7) $f_{H_{11},\ldots,H_{n_t,n_r}}$ is the probability density function of the channel coefficients, the integral is over all values of $\mathbf{H}$, and the selection rule $\tilde{\mathbf{H}} = g(\mathbf{H})$ is important in determining the integrand. If the optimum selection rule for $E\{C_M\}$ will select a particular set of transmit and receive antennas for a particular instance of $H_{11}, \ldots, H_{n_t,n_r}$ from (7), then due to symmetry this same selection will also occur several more times as we run through all the possible values of $H_{11}, \ldots, H_{n_t,n_r}$. Thus assume that terms with $|H_{\hat{i}\hat{j}}|^2 = a$ and $|H_{\hat{i}\hat{j}'}|^2 = b$ in (4) are large enough to cause the corresponding antennas to be selected by the selection criterion trying to maximize (4) for some set of $H_{11}, \ldots, H_{n_t,n_r}$. Then due to the symmetry $(\tilde{H}_{ij}, \tilde{H}_{ij'}^*) = (\sqrt{a}e^{j\phi_a}, \sqrt{b}e^{j\phi_b})$, $(\tilde{H}_{ij}, \tilde{H}_{ij}^*) = (\sqrt{a}e^{j\phi_a}, -\sqrt{b}e^{j\phi_b})$, $(\tilde{H}_{ij}, \tilde{H}_{ij'}^*) = (-\sqrt{a}e^{j\phi_a}, \sqrt{b}e^{j\phi_b})$ and $(H_{ij}, H_{ij'}^*) = (-\sqrt{a}e^{j\phi_a}, -\sqrt{b}e^{j\phi_b})$ will all appear in (7). Since each of these four possible values appear for four equal area (actually probability) regions in channel coefficient space, a complete cancellation of these terms results in (7). In fact this leads to (7), and the other cross terms like it, averaging to zero. Thus if we use the selection approach that will maximize $E\{C_M\}$ for both signaling alternatives we find $E\{C_M\} = E\{C_1\}$ and we note that this is the best we can do for $E\{C_M\}$.

However, we can do better for $E\{C_1\}$. Due to the cross terms in (5) we can use selection to make $E\{C_1\} > E\{C_M\}$ by modifying the selection approach which is best for $E\{C_M\}$. To understand the basic idea, let $\tilde{\mathbf{H}}'$ denote the matrix $\tilde{\mathbf{H}}$ for a particular selection of antennas and $\tilde{\mathbf{H}}''$ denote the same quantity for a different selection of antennas. Now consider two selection approaches which are the same except the second approach will choose $\tilde{\mathbf{H}}''$ in cases where

$$\sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} |\tilde{H}''_{ij}|^2 = \sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} |\tilde{H}'_{ij}|^2$$

$$\sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} \sum_{j'=1,\, j'\neq j}^{n_{st}} \tilde{H}''_{ij}\tilde{H}''^*_{ij'} > 0$$

$$\sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} \sum_{j'=1,\, j'\neq j}^{n_{st}} \tilde{H}'_{ij}\tilde{H}'^*_{ij'} < 0.$$

Assume the first selection approach is the one trying to maximize $E\{C_M\}$ so it will just select randomly if

$$\sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} |\tilde{H}''_{ij}|^2 = \sum_{i=1}^{n_{sr}} \sum_{j=1}^{n_{st}} |\tilde{H}'_{ij}|^2$$

since it ignores the cross terms in its selection. From (4) and (5), the second selection approach will give $C_1 > C_M$ for each event where the selection is different. Since the probability of

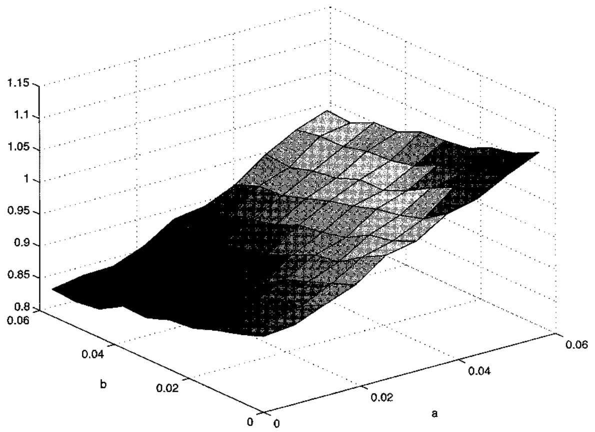

Fig. 1. Average mutual information $E\{C\}$ versus b and a with selection and -9-dB SNR.

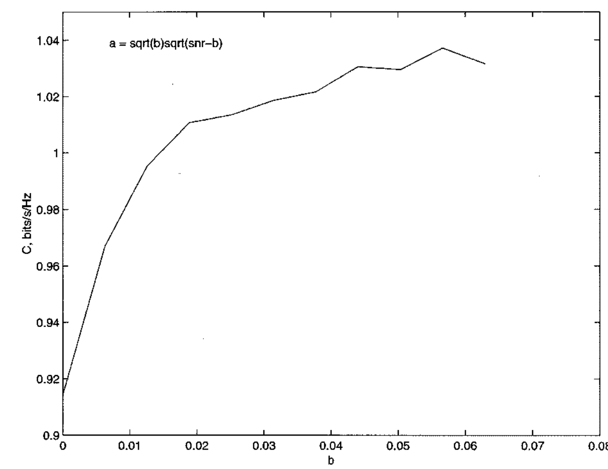

Fig. 2. Average mutual information $E\{C\}$ versus b for $a = \sqrt{b}\,\sqrt{\rho - b}$ with selection and -9-dB SNR. Best performance occurs at $b = a = \rho/2$.

the event that makes the two approaches different is greater than zero under our assumed model, then the second antenna selection approach, which we call the modified approach, will lead to $E\{C_1\} > E\{C_M\}$.

Thus if we apply this improvement procedure to the selection approach that is optimum for $E\{C_M\}$ we will get improvement for the single stream approach so that $E\{C_1\} > E\{C_M\}$. It is key that C_M has no cross terms, so it can't be improved in this way. Thus it follows that the single stream approach can be made to be better than the approach using $\mathbf{Q} = (\rho/n_{st})\mathbf{I}_{n_{st}}$ by using the proper selection approach. Clearly the optimum selection scheme for $E\{C_1\}$ will be at least as good or better so it must also give $E\{C_1\} > E\{C_M\}$ if both approaches use optimum selection. □

One might wonder why the result in Theorem 1 differs from those in [8]. A very short explanation is that the effective statistics of $\tilde{H}$ are generally no longer complex Gaussian after selection, which is a needed condition for some of the results in [8]. The reason is that selection is not a linear operation and the resulting nonGaussianity after selection is well known from the study of order statistics [10]. Numerical results also indicate there are weak signal cases where $\mathbf{Q} = (\rho/n_{st})\mathbf{I}_{n_{st}}$ does not lead to best performance when antenna selection is employed. The average mutual information with general $\mathbf{Q}$ and selection is

$$E\{C\} = \Psi(\mathbf{Q}, \tilde{\mathbf{H}}) = E\left\{\log_2\left(\det\left(\mathbf{I}_{n_r} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^H\right)\right)\right\}. \tag{8}$$

The single stream MIMO approach corresponds to using a constant matrix $\mathbf{Q}$ with all entries equal to ρ/n_t. Using a $\mathbf{Q}$ of this form can sometimes provide better performance than can be obtained using $\mathbf{Q} = (\rho/n_{st})\mathbf{I}_{n_{st}}$ when optimum selection is used in both cases. This is illustrated by the results in Figs. 1 and 2 for a case with $\rho = -9$ dB, $n_t = n_r = 8, n_{st} = n_{sr} = 2$. Fig. 1 shows a plot of $\Psi(\mathbf{Q}, \tilde{\mathbf{H}})$ versus the scalars b and a when

$$\mathbf{Q} = \begin{pmatrix} b & a \\ a & \rho - b \end{pmatrix}. \tag{9}$$

In interpreting Fig. 1 we recall that power is fixed so $0 \le b \le \rho$ but due to symmetry only $0 \le b \le \rho/2$ need be considered. Furthermore, due to the definition of a we find $a \le \sqrt{b}\sqrt{\rho - b} \le \rho/2$ which means only points with $a \le \sqrt{b}\sqrt{\rho - b}$ are valid. Thus Figs. 1 and 2 indicate best performance is achieved with $b = \rho/2$ and $a = \rho/2$ which corresponds to the single stream approach. The results are different if we consider larger ρ [11] where we find $b = \rho/2$ and $a = 0$ gives best performance.

IV. Conclusions

The ergodic capacity of MIMO with antenna selection has been studied for cases with limited feedback from the receiver to the transmitter. In particular, the optimum signaling scheme has been considered. It was shown that the optimum signaling for a single, isolated MIMO link, with antenna selection, is generally different from that which is optimum without antenna selection. In cases with interference the improvement obtained from using the true optimum signaling approach tends to increase for larger interference [11]. Furthermore the optimum approach for cases without antenna selection tends to be optimum for fewer cases as the interference is increased.

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